In presenting this dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the Dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

AN INVESTIGATION OF COMMERCIAL-AIRCRAFT LANDING-GEAR WHEEL-ASSEMBLY SERVICE-LIFE IN ORDER TO DEVELOP INVENTORY DECISION MODELS FOR SPARE ASSEMBLIES AND TIRES

A THESIS

Presented to

the Faculty of the Graduate Division

by

Robert Stephen Friedman

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Industrial Engineering

Georgia Institute of Technology

May, 1962

9 8
12 R

AN INVESTIGATION OF COMMERCIAL-AIRCRAFT LANDING-GEAR WHEEL-ASSEMBLY SERVICE-LIFE IN ORDER TO DEVELOP INVENTORY DECISION MODELS FOR SPARE ASSEMBLIES AND TIRES

APPROVED:

Harold E. Smalley

Date approved by Chairman: March 28, 1962

ACKNOWLEDGMENTS

The guidance received from Dr. Harold E. Smalley in the course of this study is very deeply appreciated. Appreciation is also extended to Dr. Joseph J. Moder, Jr. and Dr. John H. MacKay who served on my Advisory Committee, and to Dr. David C. Ekey of the University of Richmond, Richmond, Virginia.

The cooperation received from Delta Airlines and Braniff Airways in providing data and information is gratefully acknowledged, since without their cooperation this study could not have been carried out.

Sincerest thanks must also go to my wife, Carolyn Friedman, whose encouragement and patience played a major role in my completing this study and the other requirements for the Master of Science degree.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

General Problem and Objective. --Commercial airlines stock various spare aircraft parts and assemblies at airport stations, which are designated as maintenance bases or line-maintenance-stations. The spares at the maintenance bases are utilized primarily for scheduled aircraft maintenance removals, while the line maintenance station spares are stocked primarily to minimize the number and duration of flight delays caused by non-scheduled removals. A non-scheduled removal could cause a long flight delay if the necessary spare had to be shipped from another maintenance station or base.

One type of spare which is stocked at both maintenance bases and stations is the landing-gear wheel-assembly. This assembly consists of a wheel and mounted tire, and requires replacement whenever the tire or wheel is not in serviceable condition. For the purpose of this study, the unit of inventory at the line maintenance stations will be the complete wheel assembly. Unmounted wheels and tires are not stocked at line stations because of the excessive equipment and personnel costs associated with tire mounting. Maintenance bases have both complete wheel assemblies and unmounted wheels and tires. The unmounted components (i. e., wheels and tires) are distributed among the various stages of overhaul and repair.

The objective of this study was the formulation of inventory decision models which can be used to develop inventory policies for wheel assemblies and tires. The inventory demand parameter was the number of aircraft landings per assembly removal. This parameter was investigated for the total airline system, and was found to be correlated to the service-life obtained from the tire component of the assembly. Service-life characteristics of new and retreaded tires were studied, and the mean service-life of both was found to be correlated to seasonal climatological temperature fluctuations.

Consideration in the inventory models was given to demand fluctuations, flight delay costs due to inventory shortages, and inventory carrying costs. Variations in the inventory demand parameter were studied for the total airline system, and for the individual maintenance bases and line maintenance stations. All inventory replenishment lead times were assumed to be constant.

While the development of the inventory models was the primary motivation for this study, it was necessary to investigate the complex operational factors related to tire service-life in order to develop the inventory demand function. Reliable technical information relative to tire service-life was not available in the literature, and it was found that airlines currently evaluate tire performance by the use of individual experience factors.

Choice of Assembly to be Studied. --The data used to develop these models were obtained from Delta Airlines, Atlanta, Georgia, and Braniff Airways, Dallas, Texas. The specific assembly studied was the Convair 340/440 main-landing-gear assembly. The Convair 340/440 is a twin-engined, piston-powered, 44-passenger aircraft which has two wheel assemblies installed on each of its two main landing gears. In addition to the four main wheel assemblies, it also has two nose wheel assemblies installed on the nose landing gear.

The Convair 340/440 main wheel assembly was selected for study because the airlines providing data operate a large number of this type of aircraft over flight routes which require frequent landings. These characteristics provided a large amount of data relative to wheel assembly replacement, since assembly service-life is primarily a function of the number of aircraft landings to which the assembly has been subjected. Other types of aircraft considered were found to be utilized in lesser numbers and over flight routes which required fewer landings.

Discussions with representatives of Delta Airlines revealed that they presently use a "rule of thumb" combination of historical demand and estimates made by line-maintenance-station foremen to determine inventory levels of spare assemblies. This study investigates several of the factors affecting assembly demand, and the models developed can be utilized to adjust inventories as operational conditions vary.

Total airline investment in spare wheel components and assemblies is substantial. In March of 1960, Delta Airlines operated 28 Convair 340/440 aircraft which provided service to 44 different airports. Forty-three spare Convair 340/440 main wheel assemblies were stocked in varying quantities at 25 line maintenance stations. There also were 15 spare assemblies stocked at the Atlanta maintenance base which were used to replenish line-maintenance-station stocks. In addition to the above mentioned assemblies, there were 21 wheels and 106 unmounted tires in various states of transit, overhaul or retreading.

A Convair 340/440 main wheel assembly represents an investment of \$360.00, with the wheel costing approximately \$245.00 and the tire costing \$115.00. Thus Delta's total investment in spare components and assemblies for this one type of spare assembly is approximately \$38,000.00. Delta Airlines also operates the DC-3, DC-6, DC-7, DC-8, and Convair 880 passenger type aircraft. These aircraft require eleven different types of wheels and twelve different types of tires to be stocked in varying quantities (the DC-6 and DC-7 nose wheels are interchangeable, but the tires require different ply-ratings). The trend toward jet powered aircraft will add heavily to component and assembly investment (e. g., the \$725.00 cost of a jet DC-8 main wheel and \$360.00 cost of a DC-8 main tire yields an assembly cost of \$1,085.00). The problem of optimal distribution of spares will also become more complex if the current trend of adding new stations to routes is continued.

Although this study is limited to data from the Convair 340/440 main wheel assembly, many of the conclusions and techniques may be applicable to other types of wheel assemblies.

Aircraft Tire Characteristics.--Aircraft tires that can withstand high speeds and heavy impacts are absolutely essential to the safe operational performance of all types of aircraft. Aircraft tires principally perform three functions. These are as follows:

1. Cushion the shock of landings.
2. Permit the airplane to be brought safely to a stop.
3. Facilitate take-offs.

The tire consists basically of the rubber tread and sidewall, cord body, and wire beads. The tread rubber on the outer circumference of the tire serves as the wearing surface and along with the sidewall protects the cord body from cuts, bruises and moisture. The cord body generally consists of an even number of rubberized-nylon cords which are sometimes reinforced by the addition of breakers or inserts directly under the tread. The steel wire beads are necessary to hold the tire carcass on the wheel rim because of the centrifugal forces built up at high speeds. Figure 1 is a cut-away view showing the typical construction of a tubeless aircraft tire.

The Convair 340/440 main tire is a 12.50 x 16, 12 ply-rating tire. The term ply-rating is used to identify a given tire or casing with its maximum recommended load when used in a specific type of service. It is an index of the tire or casing strength and does not necessarily

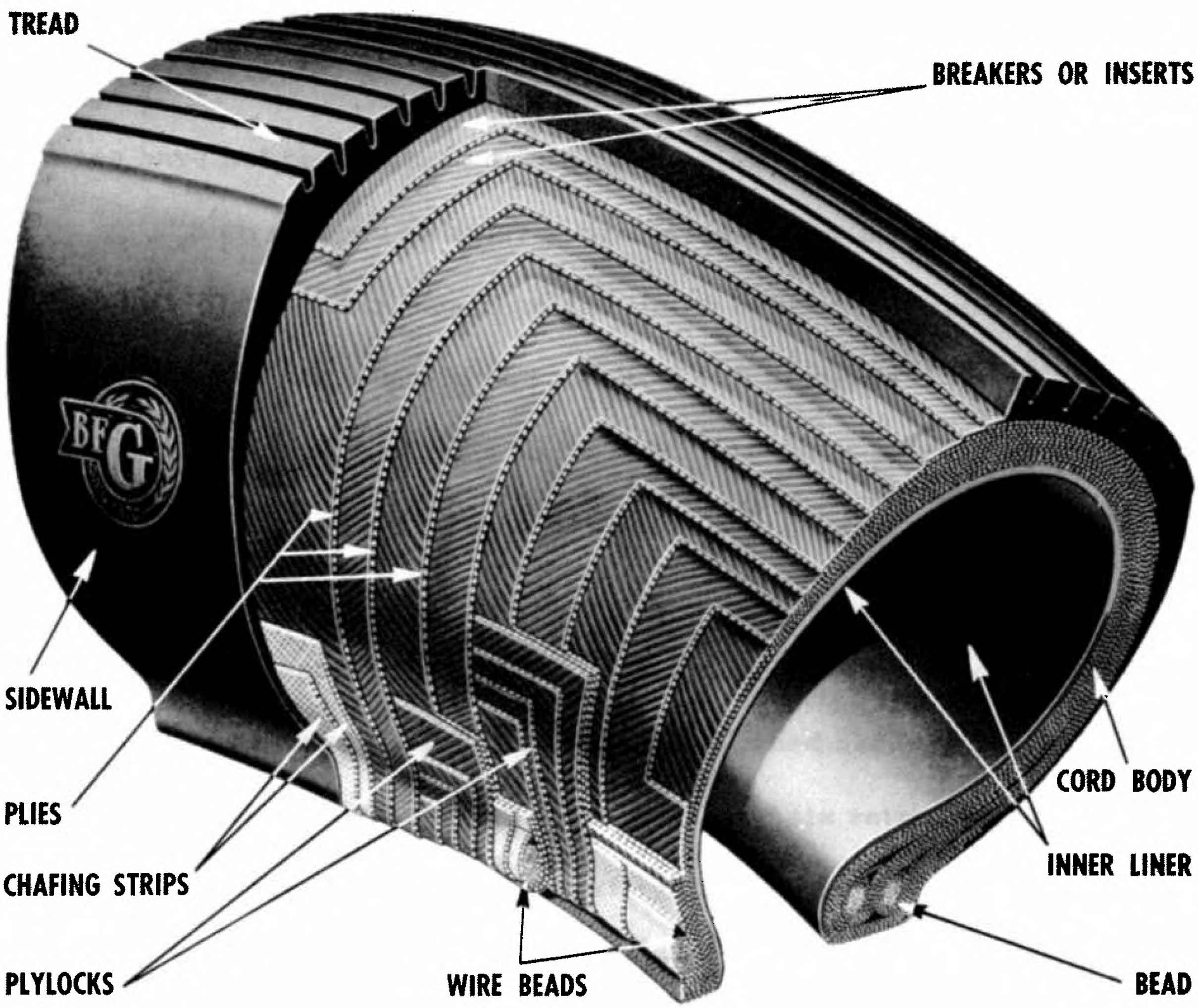


Figure 1. Cut-away View Showing Typical Construction of a Conventional Tubeless Aircraft Tire (Photograph by Permission of B. F. Goodrich Rubber Company, Akron, Ohio)

represent the number of cord plies (1)[1]. The normal inflation pressure employed by Delta for this tire is 66 pounds per square inch.

Airline Utilization of Retreaded Tires.--In order to reduce tire costs airlines have universally accepted the retreaded tire. However, the airlines have not been able to agree upon a standard policy with respect to the maximum number of times a tire carcass may have a retread applied. In 1957 three airlines reported limits of thirteen, ten and four on the number of times a tire carcass could be retreaded. Some other airlines specified no particular limit, but allowed the retreading agency to make decisions based entirely on the condition of each individual tire carcass (2).

Delta has established a limit of six retreads on the Convair 340/440 main-tire carcass. This limit was established because past performance had shown that several tires with over six retreads had serious failures in service and presented a safety problem. The data in Braniff's monthly Tire Operation Summary for the Convair 340/440 main tire showed that only one tire with over six retreads was used during the period January 1957 through October 1960 by Braniff. Because of these facts none of the data used to investigate retreaded-tire service-life were obtained from tires with over six retreads.

Several airlines have reported that service received from

---

[1]Numbers in parentheses refer to references listed in bibliography.

retreaded tires is as good or better than service received from new tires (3, 4). The economic significance of the retreaded tire is shown in the tire-cost report of a major airline for a twelve-month period, which was prior to the introduction of jet aircraft. This report stated that 1,026 new tires were purchased at an average cost of $151.00 and 4,033 retreaded tires at an average cost of $40.50. From these data the utilization of retreaded tires saved this airline approximately $450,000.00 in a one-year period (5).

The retreaded tire is becoming of more economic significance as heavier and faster-landing jet aircraft are brought into wider use. These aircraft have almost twice as many tires as piston-type aircraft and the individual tire costs are much greater. Tires for jet aircraft are built for high loading and very high speeds. These high speeds and associated centrifugal forces have required a smaller tread depth, which severely lowers tire service-life. In January of 1960, American Airlines reported that the jet Boeing 707 main-tire carcass was being retreaded a maximum of three times. Tests and service experience were the determining factors in establishing these limitations for American. American was obtaining an average of 82 landings from new tires and an average of 70 from retreaded tires; however, American anticipates these will be changed as more experience with the Boeing 707 is gained (6).

The economies and reliable service records associated with retreaded tires have caused airlines to utilize them in great quantities.

During the period 1954 through 1957 only 20 instances, out of 15 million take-offs and landings, of a retreaded-tire failure during take-off, landing, or taxiing were reported (7). This record, combined with the efforts extended by retreading agencies to improve their product, will very likely lead to an increase of retreaded-tire utilization in the future.

Inventory Problem. --There recently has been a considerable amount of literature written in regard to the inventory problem. Inventory theories have two basic concepts (8). The first is the "distribution function," which summarizes past demand. The second is the "loss function," which is developed from the associated costs of ordering, storage and shortage.

An approach to a business management decision problem can be stated by the following steps (9):

1. A model expressing a set of assumed empirical relations among a set of variables.
2. A subset of decision variables whose values are to be chosen by the firm or other decision making entity.
3. An objective function of the variables in the model, a function such that a higher value represents a more desirable state of affairs from the viewpoint of the firm.
4. Computing methods for analyzing the effects of

alternative values of the decision variables on the objective function.

The above approach is used in the development of the inventory models in this study.

The principal inventory theory concept studied is the "distribution function." Less extensive study is made of the "loss function" and the associated costs of carrying inventory and inventory shortages.

CHAPTER II

EXPERIMENTAL PROCEDURE

Delta Airlines and Braniff Airways were extremely cooperative in providing data relative to the "distribution function" and "loss function" for the inventory item studied.

Data from Braniff Airways.--The data from Braniff were obtained from its monthly "Tire Operation Summary" for the period January 1957 through October 1959. These summaries contained service-life data on approximately 85 per cent of the wheel assemblies removed. The Chief Engineer of Braniff Airways explained why some removals are not contained in the summaries. Frequently assemblies with worn tires were removed before an aircraft went into a flight pattern requiring numerous landings outside the Braniff system--a flight pattern which did not return the aircraft to an equipped maintenance base. A flight pattern of this type occurs when a Braniff aircraft is flown on an interchange route by Eastern Airlines from Memphis to Miami, and return. The removed assemblies are held as spare units and are installed on aircraft going back into the Braniff system, where maintenance service is more readily available. These prematurely removed assemblies are not recorded on the summary and account for the largest part of the missing data. The remainder of the unreported removals are attributed to the receipt of overhaul reports on tires and wheels for which records were not available. Since the summaries contained service-life data on approximately 85 per cent of the Braniff assembly removals, it was assumed that these data were representative of the service-life of all Braniff removals. This problem of incomplete records did not exist for Delta data.

The Braniff summary classified assembly removals by aircraft type and aircraft position (i. e., nose, main, and tail). The measure of service-life was the number of landings obtained from the removed assembly.

Removal cause classifications were as follows:

1. Minor defects
   a. Cracked tire tread
   b. Cracked tire sidewall
   c. Tire out of balance
2. Major defects
   a. Tire splice parting
   b. Tire ply separation
   c. Loose tire undertread
   d. Tire tread separation
   e. Tire bead failure
3. Operational failure
   a. Tire skid burns

b. Tire tread cut

c. Tire sidewall cut

d. Flat spot on tire tread

e. Tire shoulder scuff

f. Wheel failure

4. Wear

The manufacturer of the tire, the number of times the tire was retreaded, and the name of the retreading agency were also given. The number of times a tire has been retreaded will be referred to in this study as the retread stage. Several new tire manufacturers and tire retreading agencies were represented in the summary. For example, the summary would show that three tires in the second retread stage were removed because of wear, after averaging 395 landings. These tires were originally manufactured by the Firestone Tire and Rubber Company and were retreaded by the General Tire Service. The summary does not show the service-life of each of the three tires, but only the average of the three.

Two types of data were obtained from the Braniff summary. The first type of data was obtained from removals which were shown by the summary as individual removals, rather than averages of two or more. These data are presented in Appendix A, and are classified according to new tire and retread stage. Appendix A presents service-life data for 11.9 per cent of all the new-tire removals and 34.7 per cent of all retreaded-tire removals in the summary. The higher percentage of retreaded tires was because the six retread stages and the independent retreader classifications provided more opportunity for retreaded tires to be shown as individual items. The reason for obtaining the data in Appendix A was to provide data for the computation of estimates of the service-life variances for new and retreaded tires.

The second type of data was obtained by combining the service-life data of individual removals, and averages of two or more removals. This procedure provided an estimate of the mean service-life of tires for the specific classification (i. e., month, manufacturer, retread stage, etc.) studied.

Data from Delta Airlines.--Most of the data in this study were obtained from Delta Airlines. These data were for the period April 1957 through April 1960. "Aircraft Wheel Record Cards" for each of the four main-wheel assemblies on each of the 28 Convair 340/440's provided service-life data in hours on 3,611 assembly removals. These data are shown in Appendices A and B. The "Aircraft Wheel Record Cards" did not provide any information regarding the station making the change, the removal cause, or whether the tire was new or retreaded. To get this latter information it was necessary to utilize the "Wheel Assembly Unit Removal Tags." These tags showed the station making the replacement; the reason for the removal; the date of the removal; the flight the aircraft was on; the aircraft number and wheel position; whether the tire was new or retreaded;

and the wheel numbers of the installed and removed assemblies. In instances where the "Wheel Assembly Unit Removal Tags" did not provide all the data needed, the "Daily Aircraft Log Sheets" were checked to see if they contained the needed data. These daily logs provided data on the station making the change, the date of the change, the aircraft wheel position, and frequently the removal cause.

Since the above data were related to the aircraft and wheel, it was necessary to utilize the "Tire Record Cards" to investigate the service-life performance of a specific tire through the new and succeeding retread stages. The "Tire Record Cards" provided a chronological history of each tire in the system, and were classified by the tire manufacturers. These cards showed the tire serial number; the retread stage of the tire carcass; the number of the wheel on which the tire was mounted; the dates the wheel assembly was installed and removed; and the cumulative service-life of the new tire and succeeding retreads.

The service-life was determined by the date the wheel assembly was installed on the aircraft and the date it was removed. A daily "Aircraft Utilization Report" was prepared which showed the number of flight hours each aircraft had at midnight on the day of the report. The service-life obtained from a specific wheel assembly was determined by subtracting the total flight hours on the date of installation from the total hours on the date of removal. The "Aircraft Wheel Record Cards," "Wheel Assembly Unit Removal Tags," and "Tire Record Cards" were all used in the accumulation of data.

The following is an example of the data provided by these records. Firestone tire serial number 0435172E with three retreads was mounted on wheel number 183 and put into inventory. On November 28, 1958, this wheel assembly was mounted on aircraft number 422 in the number three position. On February 20, 1959, this assembly was removed at Memphis due to the tire being worn. The service-life obtained from this assembly during the period November 28, 1958, to February 20, 1959, was 478 hours. This assembly was returned to the Atlanta maintenance base and disassembled. The tire had a fourth retread applied and the wheel was overhauled. After wheel number 183 was overhauled it had another serviceable tire mounted on it. This assembly was put into inventory and was installed at Evansville on aircraft number 406 in the number four position on April 7. The Firestone tire with four retreads was mounted on wheel number 95, which was installed on aircraft number 420 in the number two position at Chicago on April 6. On May 1 this assembly was removed at Birmingham because of a flat tire after 200 hours of service-life. This process continues until the tire is scrapped because the carcass becomes unsuitable for retreading, or until the retread stage limit has been reached. Wheel service-life is much longer than tire service-life, and is independent of tire service-life.

The Delta service-life data were converted into landings by the use of adjusted monthly "landing factors." The term "landing factor" refers to the average number of aircraft landings per flight hour. Records were available which showed the number of hours flown per month and the number of landings made per month by the Delta fleet of Convair 340/440 aircraft. The ratio of monthly landings to hours yielded monthly "landing factors." The adjusted monthly "landing factor" in the ith month was obtained by taking the average of the "landing factors" in the i-1 and ith months. The adjusted monthly "landing factors" were used, because the wheel assembly studied remains on an aircraft for approximately one month.

The monthly "Delta Space Availability Chart" was utilized to determine the monthly number of all station landings which terminate flights. This chart showed the originating, non-terminating, and terminating stations for each flight number.

A daily "Flight Delay Report" was utilized to obtain data on delays caused by wheel assemblies. This report included delays caused by lack of an available replacement spare, and delays due to the time involved to perform the removal.

Climatological data on temperature were obtained from the United States Department of Commerce. Data on monthly "record mean" temperatures for Atlanta, Georgia; Dallas, Texas; and Chicago, Illinois, were obtained. These monthly "record mean" temperatures are based on historical data and were utilized as the best estimate of the average monthly temperature of the aforementioned three cities. Data on actual average monthly temperatures for Atlanta, Georgia, for the period December 1956 through April 1960 were also obtained.

Effect of Climatological Temperature Fluctuations on Tire Service-Life. --In the interest of forecasting tire service-life and the related demand measure of landings per removal, it was desirable to locate a factor which correlated with tire service-life.

Seasonal temperature fluctuation was the principal factor found in this study to correlate with the service-life of new and retreaded tires. The "landing gear engineer" at Delta stated that mean tire service-life was 10 to 15 per cent less in the summer months than in the winter months, a factor which motivated the investigation of the seasonal temperature fluctuation factor.

Seasonal temperature fluctuations were considered to be an important factor in forecasting wheel-assembly demand since tire-tread wear increases directly with a rise in temperature (10). The rise in tire-operating temperature due to friction heat buildup and ambient temperatures decreases both tire-carcass and tire-tread service-life.

Aircraft tires develop an internal temperature rise because of the hysteretic nature of tire materials as the tires roll along the ground under load. The fact that rubber is a very poor conductor of heat causes the tire to develop hot spots. These hot spot temperatures depend upon the duration of the roll and the speed. Hot spots shorten the life of the carcass because the carcass fabrics currently in use are affected considerably by high temperatures. For example, nylon cord, which is used almost exclusively in high performance aircraft tires, will retain most of its original strength when exposed to high temperatures for limited periods of time. However, continued exposure to high temperatures will give permanent degradation (11). These temperature rises also weaken the strength of the tread rubber and its adhesion to the tire carcass. Centrifugal forces tend to pull the tread away from the tire carcass at high speeds, and this is more likely to occur when the temperature rises caused by the rolling of the tire under load has considerably weakened the tread adhesion. For this reason there has been a tendency to use thinner treads which have the effect of reducing the centrifugal force and of providing better cooling of the tire hot spots.

Ambient temperature means the temperature in the immediate environment of the tire. The ambient temperature is affected by the proximity of the tire to the engines, the braking heat, and the runway temperature. During landings and taxiing, the brake heat travels into the wheel structure and often overheats the tire beads. However, the most serious problem facing the aircraft designers and aircraft tire designers is the effect of temperature rise due to the air friction of high-speed aircraft. Some tests have been made to determine the temperature limit and the duration of exposure for tires of present rubber and fabric construction. These tests have indicated that the less the duration of exposure to heat, the higher the ambient temperature the tire will stand. Tests have been made on tires exposed to temperatures ranging from 285°F. to as high as 500°F. for various exposure times (12). This technical information further indicated that climatological temperatures might be a significant predictor of tire service-life.

Use of Monthly Temperatures in Atlanta, Georgia for Correlations with Tire Service-Life.--The climatological temperatures used in correlations with tire service-life were the average monthly temperatures for Atlanta, Georgia. It was assumed that the temperature fluctuations in Atlanta were representative of the varied temperatures in the geographical areas served by Braniff and Delta. Braniff utilizes the Convair 340/440 on routes which extend from Miami, Florida, in the south to Minneapolis, Minnesota, in the north. Delta's routes extend from the Caribbean Sea area in the south to Detroit, Michigan in the north. The use of Atlanta temperatures was further based on the assumption that the aircraft were rotated at random throughout the entire route system. This assumption was substantiated by oral discussion with personnel in the aircraft scheduling department of Delta. The choice of Atlanta temperatures was also influenced by the fact that Atlanta is the headquarters of Delta operations.

Correlation of Inventory Demand Measure and Climatological Temperature Fluctuations.--The inventory demand parameter, landings per assembly removal, was found to be a direct function of the tire service-life. Therefore, since tire service-life was an inverse function of climatological temperature fluctuations, the inventory demand parameter was also an inverse function of climatological temperature fluctuations.

A procedure to forecast the demand parameter was developed on the basis of the monthly adjusted Atlanta mean temperatures and the extent of new and retreaded-tire utilization. Differences were found between new-tire service-life and retreaded-tire service-life, which caused the inventory demand parameter to be a function of the extent of new and retreaded-tire utilization.

Two correction factors were developed which improved the tire service-life estimate of the inventory demand parameter. One factor was a function of the monthly Atlanta mean temperature in the month to be forecasted and in the previous month. The other factor was based on the assumption that over a long period of time the sum of the tire service-life overestimates and underestimates of the inventory demand parameter would tend toward zero.

Basis for Dividing the Calendar Year into Three Seasons. --The calendar year was divided into three seasons to facilitate some of the investigations of tire service-life and the daily assembly demand distribution. These seasons were formed by combining months with similar temperatures. The division was based on the "record mean" monthly temperatures presented in Table 1. These data were furnished by the United States Department of Commerce for three cities in the United States.

The three cities chosen to represent various climatical conditions were Atlanta, Georgia; Dallas, Texas; and Chicago, Illinois. The "record mean" temperature is based on historical average monthly temperatures and was assumed to be the best estimate of average monthly temperatures.

It was assumed that the temperature in the i-1 month will frequently affect the service-life of removals in the ith month, because the assembly investigated remains installed on an aircraft for approximately one month. Therefore, averages of the "record mean" temperature in the i-1 month and the "record mean" temperature in the ith month were used in the development of the seasonal division of the calendar year. Table 2 presents monthly temperatures which are an average of the "record mean" temperature of the i-1 and ith months for the above three cities. For example, the Chicago average "February" temperature shown in Table 2 is equal to the (January "record mean" temperature plus February "record mean" monthly temperature) divided by two.

The months December, January, February, and March are designated Season I; April, May, October, and November are designated Season II; and June, July, August, and September are designated as Season III. These three seasons were used in analysis of variance investigations of tire service-life and in chi-square tests of the fit of Poisson distributions to the daily assembly demand distributions.

Daily Distribution of System and Station Assembly Demand. --A

Table 1. Monthly Record Mean Temperatures (m_i) for Atlanta, Georgia; Dallas, Texas; and Chicago, Illinois

Month	Atlanta	Dallas	Chicago
January	43.7	45.8	25.0
February	46.0	49.8	27.1
March	52.7	56.7	36.3
April	61.3	65.2	47.5
May	69.8	72.9	57.9
June	76.7	81.2	68.0
July	78.7	84.8	73.6
August	77.9	84.8	72.3
September	73.3	78.0	65.5
October	62.9	67.9	54.4
November	52.0	55.7	40.2
December	44.8	47.9	29.6
Annual	61.6	65.9	49.8

Table 2. Average of the Record Mean Temperatures in the ith Month and (i-1) Month for Atlanta, Georgia; Dallas, Texas; and Chicago, Illinois

	ith Month	Atlanta	Dallas	Chicago
	December	48.40	51.80	34.90
Season I	January	44.25	46.85	27.30
	February	44.85	47.80	26.05
	March	49.35	53.25	31.70
	April	57.00	60.95	41.90
Season II	May	65.55	69.05	52.70
	October	68.10	72.95	59.95
	November	57.45	61.80	47.30
	June	73.25	77.05	62.95
Season III	July	77.70	83.00	70.80
	August	78.30	84.80	72.95
	September	75.60	81.40	68.90

maintenance official of Delta Airlines stated that maintenance base and line maintenance station spare-assembly inventories are replenished daily. This one-day replenishment lead time resulted in an analysis of the system and station daily assembly demand distributions.

The observed daily distributions of system and station assembly removals were found to be approximated by Poisson distributed populations with the same means as the observed data. It was found, however, that the inventory demand parameter (landings per removal) varied from station to station. The station inventory demand parameter was found to be related to the maintenance capability of the station, the proportion of station landings resulting in flight terminations, and management policies.

Flight Delays Due to Wheel Assemblies.--A study of flight delays caused by replacing unserviceable assemblies was conducted. The following factors were studied:

1. Service-life obtained from the assemblies causing the delays.
2. Causes of these assembly removals.
3. Comparison of number of delays caused by new and retreaded tires.

Development of Inventory Decision Models.--Inventory decision models for assembly and tire inventory policies were developed. The decision models are based on the results of the tire service-life analyses and the analyses of the assembly demand distribution.

The inventory decision models are based on Delta data and the characteristics of the Delta inventory system. Three inventory problems were considered, which are as follows:

1. Assembly-stocking policies for maintenance bases and line maintenance stations.
2. Assembly-stocking policies for the Atlanta secondary inventory.
3. Tire-stocking policies at the Atlanta maintenance base.

The symbols used most frequently in the analyses of tire service-life, the analyses of the assembly demand distribution, and the development of the inventory decision models are shown in Table 3.

Table 3. Symbols Used in the Analyses of Service-Life and Inventory Model Construction

a	= a th retread stage, where a = 1, 2, 3, 4, 5, or 6
A	= forecasted weekly assembly demand (number of assemblies)
$\overline{A}$	= forecasted weekly mean assembly demand (number of assemblies)
B	= actual weekly assembly demand (number of assemblies)
$\overline{B}$	= actual mean weekly assembly demand (number of assemblies)
$\overline{c}_i$	= adjusted monthly mean service-life of all assembly removals (hours or landings)
C_1	= inventory carrying cost (dollars)
C_2	= inventory shortage cost (dollars)
C_A	= total cost of wheel assembly (dollars)
$\overline{g}_i$	= monthly mean service-life of all assembly removals (hours or landings)
G	= daily assembly demand (number of assemblies)
$\overline{G}$	= mean daily assembly demand (number of assemblies)
H_i	= monthly number of aircraft flight hours
i	= ith month
I_A	= forecasted required-assembly inventory level (number of assemblies)
I_B	= actual required-assembly inventory level (number of assemblies)
J_i	= monthly forecasted number of scrapped tires (number of tires)
K_d	= proportion of delays caused by retreaded tires

(Cont.)

Table 3. (Cont.) Symbols Used in the Analyses of Service-Life and Inventory Model Construction

K_r	= proportion of all tires that are retreaded tires
L_i	= monthly number of aircraft landings
LF_i	= monthly landing factor (landings per hour)
m_i	= record mean monthly temperatures (degrees Fahrenheit)
M	= number of spare assemblies stocked at an airport station (number of assemblies)
MF_i	= adjusted monthly landing factor (landings per hour)
$\overline{n}_i$	= monthly mean service-life of new tires (hours or landings)
N_a	= number of tires in retread stage a
N_g	= number of tires (new and retreaded)
N_n	= number of new tires
N_r	= number of retreaded tires
q_i	= average monthly temperature in Atlanta, Georgia (degrees Fahrenheit)
Q_i	= monthly forecasted number of assembly removals (number of assemblies)
$\overline{r}_a$	= mean service-life of retreaded tires in retread stage a (hours or landings)
$\overline{r}_i$	= monthly service-life of all retreaded tires (hours or landings)
R_i	= monthly number of assembly removals
s_a^2	= service-life variance of retreaded tires in retread stage a
s_g^2	= service-life variance of all assembly removals

(Cont.)

Table 3. (Cont.) Symbols Used in the Analyses of Service-Life and Inventory Model Construction

s_n^2	= service-life variance of new tires
s_r^2	= service-life variance of all retreaded tires
$s_{y/x}$	= estimate of standard error of estimate for regression of y on x
t_i	= adjusted average monthly temperature in Atlanta, Georgia (degrees Fahrenheit)
T_i	= monthly forecasted required-tire inventory level (number of tires)
TEC	= total expected annual inventory cost (dollars)
U_i	= monthly required-tire inventory level (number of tires)
V_n	= proportion of new tires removed for wear
V_r	= proportion of retreaded tires removed for wear
$\bar{w}_1$	= mean service-life of new-tire wear removals performed at Atlanta or Dallas (hours or landings)
$\bar{w}_2$	= mean service-life of new-tire wear removals performed at stations other than Atlanta or Dallas (hours or landings)
$\bar{w}_3$	= mean service-life of retreaded-tire wear removals at Atlanta or Dallas (hours or landings)
$\bar{w}_4$	= mean service-life of retreaded-tire wear removals at stations other than Atlanta or Dallas (hours or landings)

CHAPTER III

ANALYSIS OF DATA AND RESULTS

Causes of Assembly Removal.--Prior to the consideration of tire service-life, a study of assembly removal causes was conducted. The following percentage classification of removal causes was obtained from the 1959 Delta data in Table 4.

1.	Tire wear	68.4%
2.	Worn spot on tire	8.6
3.	Flat or leaking tire	6.5
4.	Separation of tire tread	6.2
5.	Vibration, out of round, out of balance	4.6
6.	Tire damage (sidewall crack, cut)	3.1
7.	Worn or damaged wheel	1.4
8.	Tire blowout	.2
9.	Unknown	1.0
	Total	100.0%

The following percentage classification of removal causes for Braniff in the period November 1958 through October 1959 was obtained from the Braniff data in Table 5.

1.	Minor defects		14.3%
a.	Cracked tread	.1%	

	b. Cracked sidewall	.3	
	c. Out of balance	13.9	
2.	Major defects		2.3
	a. Loose undertread	1.0	
	b. Tread separation	1.2	
	c. Bead failure	.1	
3.	Operational failure		22.5
4.	Wear		60.9
	Total		100.0%

Comparison of the Braniff and Delta removal causes shows that the equivalent of "minor defects" totaled 7.7 per cent for the former compared to 14.3 per cent for the latter. Braniff tread separations of 2.3 per cent compared to 6.2 per cent for Delta. This latter difference was in part attributed to Delta classifying several removals as separations, which were in fact not separations. This information was obtained from a representative of the Gordy Tire Company, Atlanta, Georgia, which is the retreading agency used by Delta. Delta's 16.7 per cent for operational failure and 68.4 per cent wear removals compared to 22.5 per cent and 60.9 per cent respectively for Braniff.

No complete explanation for these variations was found to exist. However, it is possible that the numerous Delta and Braniff mechanics assigning removal causes were all not using identical removal criteria. Frequently, the assignment of a removal cause is subjective, and

Table 4. Monthly Classification of 1959 Delta Removals by Removal Causes

Month	Wear	Worn Spot	Flat or Leaking	Separation	Vibration	Tire Damage	Wheel	Blowout	Unknown	Total
January	44	7	4	8	5	4	3	1		76
February	45	5	3	8	2	4	2		2	71
March	60	6	10	9	1	2	1			89
April	87	6	6	3	2	3			1	108
May	69	11	10	9	6	4	3			112
June	75	13	7	9	10	3				117
July	83	8	10	15	3	4	1			124
August	82	8	5	4	6	7	4		2	118
September	69	9	8	6	3	3	1	1	2	102
October	86	6	4	2	5	1				104
November	69	14	7	1	5	1	2	1	2	102
December	58	11	5	1	7	1			3	86
Total	827	104	79	75	55	37	17	3	12	1,209

Table 5. Monthly Classification of Braniff Removals in Period November 1958 Through October 1959 by Removal Causes

Month	Major Defects					Minor Defects			Total
	Wear	Operational Failure	Loose Undertread	Tread Separation	Bead Failure	Cracked Tread	Cracked Sidewall	Out of Balance	
11/58	58	15	2					6	81
12/58	57	18		2				8	85
1/59	35	13						7	55
2/59	46	29		2				7	84
3/59	37	11					1	7	56
4/59	38	32		2				26	98
5/59	62	25	1	1				22	111
6/59	57	19	1	1	1			8	87
7/59	53	22	2					11	88
8/59	78	15				1	1	10	105
9/59	77	25	3	2			1	19	127
10/59	46	14	2	3				16	81
Total	644	238	11	13	1	1	3	147	1,058

different mechanics might assign different causes to the same type of removal (e. g. , a tire with a worn spot might be reported as being removed due to the vibration the spot would cause).

The 1959 Delta removal causes show that only 1. 4 per cent of the removals were due to the wheel becoming unserviceable, while the remainder of the removals were due to the tire becoming unserviceable. Since unserviceable wheels comprised such a small percentage of removal causes, it was assumed in subsequent analyses that all assembly removals were due to the unserviceable condition of the tire.

Effect of Aircraft Wheel Position on Assembly Removal Frequency. -- It was previously stated that the Convair 340/440 has four main-wheel assemblies. These assemblies are numbered one through four. Looking forward from the aircraft cockpit, numbers one and two are outboard and inboard, respectively, on the left main landing gear. Numbers three and four are inboard and outboard, respectively, on the right main landing gear.

The hypothesis that the wheel position does not affect annual assembly removal frequency was tested by a chi-square contingency test (13). The observed 1959 Delta removals are shown in Table 6 and the expected number of annual removals for each wheel position was taken to be one-fourth of the 1, 209 removals made in 1959, or 302. 25 removals. The actual chi-square value was found to be $\chi^2 = 2.33$. The critical chi-square value at the five per cent level of significance is

$\chi^2_{.05} = 7.82$. Since the actual chi-square value is less than the critical $\chi^2_{.05}$ value, there is no reason to reject the hypothesis that the wheel position does not affect assembly removal frequency. From this test it also was assumed that the wheel position does not affect new or retreaded-tire service-life.

Table 6. Monthly Classification of 1959 Delta Removals by Aircraft Wheel Position

Month	Wheel Position 1	2	3	4	Total
January	21	19	19	17	76
February	17	17	17	20	71
March	22	26	22	19	89
April	28	26	25	29	108
May	30	32	24	26	112
June	27	31	30	29	117
July	31	33	33	27	124
August	33	32	21	32	118
September	24	26	25	27	102
October	32	27	23	22	104
November	27	24	24	27	102
December	21	23	20	22	86
Total	313	316	283	297	1,209

Service-Life Distribution of New and Retreaded Tires.--The service-life distribution of new and retreaded tires was next studied. Figure 2 is a histogram showing the actual frequency distributions of the service-life which Delta obtained from 405 new tires and 967 retreaded tires.

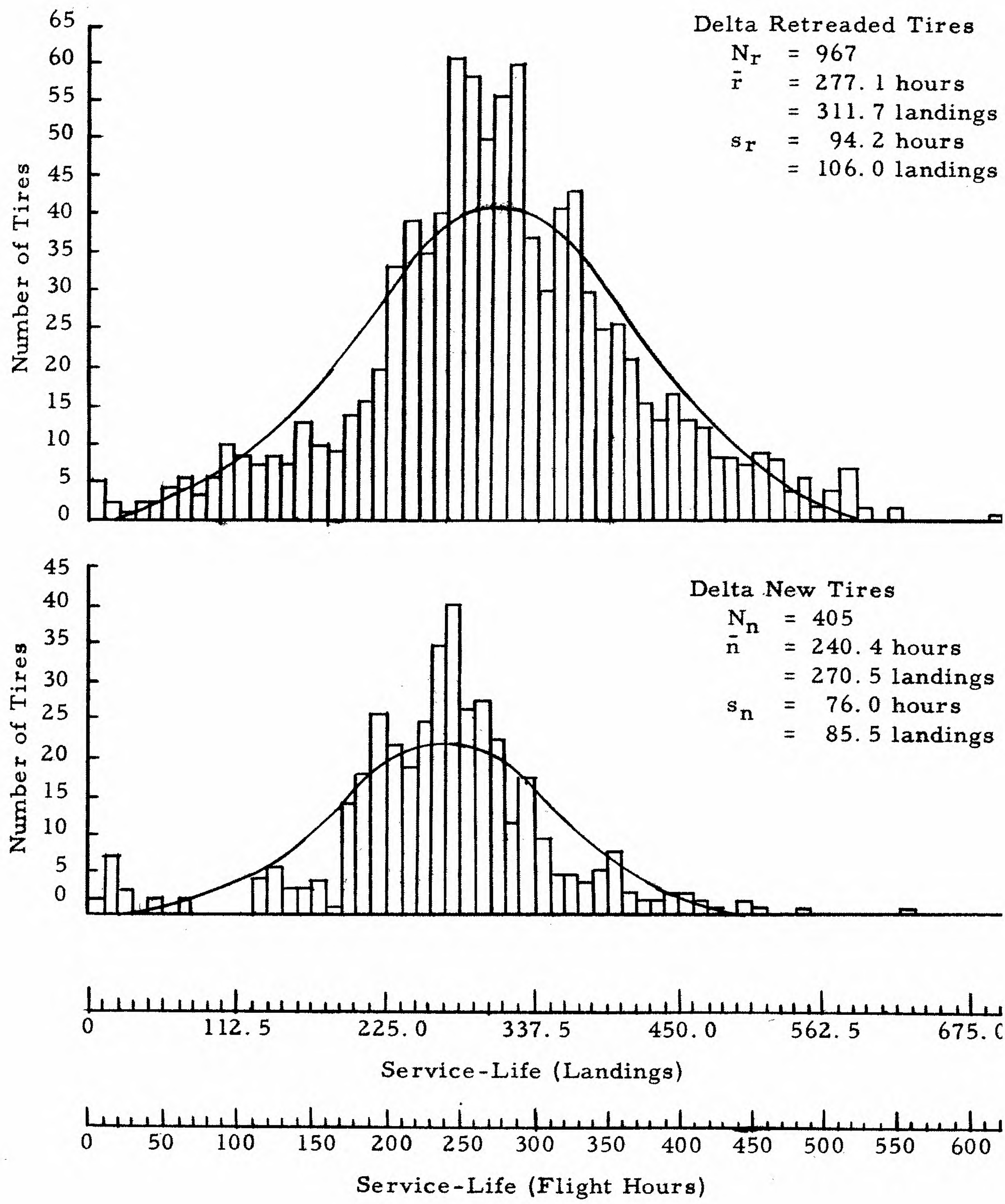


Figure 2. Actual and Theoretical Frequency Distributions of New and Retreaded-Tire Service-Life

These tires were randomly selected from removals made from April 1957 through September 1959. There are two service-life scales shown for the histogram. On the lower scale the class interval is 10 hours, while on the upper scale the class interval is 11.25 landings. The upper scale was based on the average landing factor of 1.125 landings per flight hour for the period April 1957 through September 1959.

Inspection of these histograms shows both of the frequency distributions to be unimodal. The retreaded-tire histogram has a greater variance, primarily due to a larger percentage of removals in the upper class intervals of the service-life scale. Both histograms have approximately the same percentage of removals in the class intervals between zero and 100 hours.

The theoretical normal distributions, having the same mean and standard deviation as the observed data, were compared with the actual frequency distributions. These theoretical normal distributions are also shown in Figure 2. The chi-square goodness of fit test was used to test the hypothesis that the observed distributions were from normally distributed populations (14). The actual chi-square value for the new-tire distribution was found to be

$$\chi^2 = 76.87 \qquad\qquad df = 28$$

The critical chi-square value at the five per cent level of significance is

$$\chi^2_{.05} = 41.34$$

The actual chi-square value for the retreaded-tire distribution was found to be

$$\chi^2 = 88.24 \qquad\qquad df = 41$$

The critical chi-square at the five per cent level of significance is

$$\chi^2_{.05} = 56.93$$

The actual chi-square value is greater than the critical $\chi^2_{.05}$ value for both distributions. Therefore, the hypothesis that the observed distributions are from normally distributed populations was rejected at the five per cent level of significance.

Effect of Retread Stage on Service-Life Variance.--The histograms in Figure 2 indicated that new and retreaded tires do not have equal mean service-life and variance. A study of the service-life $\bar{r}_a$, $\bar{r}$, $\bar{n}$, s_a^2, s_r^2, and s_n^2 was conducted with monthly data, seasonal data, and random data (i. e., time independent).

The first hypothesis tested was that within months the s_a^2 are homogeneous. The data used to test this hypothesis were the Delta 1959 monthly s_a^2 in Table 7. The hourly s_a^2 in Table 7 were converted into landings by multiplying by the factor MF_i^2. The factor MF_i is shown in Table 8. The statistical test used was the test developed by M. C. Bartlett, which is called Bartlett's test (15). This test uses the quantity

$$M = 2.3026 \left[n \log \left(\sum n_a s_a^2 / n \right) - n_a \log s_a^2 \right]$$

Table 7. Monthly Data (in Hours) for 1959 Delta Retreaded-Tire Service-Life by Retread Stage

Retread Stage		January	February	March	April	May	June
	N_a	15	8	16	24	32	48
1	$\bar{r}_a$	402.1	443.8	335.4	290.5	275.8	245.0
	s_a^2	11,494	1,674	16,266	10,196	6,608	5,621
	N_a	21	12	20	12	15	15
2	$\bar{r}_a$	324.4	423.2	334.7	332.3	238.9	226.4
	s_a^2	19,404	17,023	7,829	4,210	3,551	9,602
	N_a	14	18	14	17	21	15
3	$\bar{r}_a$	317.7	360.1	340.6	286.4	258.2	253.1
	s_a^2	20,086	12,330	8,017	4,894	2,562	7,383
	N_a	8	9	11	8	15	11
4	$\bar{r}_a$	448.0	294.1	318.5	284.9	232.2	285.9
	s_a^2	1,815	12,262	9,675	3,596	7,727	4,094
	N_a	5	5	6	3	4	4
5	$\bar{r}_a$	340.0	397.6	361.0	246.3	186.8	170.0
	s_a^2	21,820	6,376	41,723	2,661	12,801	15,427
	N_a		3	4		1	1
6	$\bar{r}_a$		325.0	243.2		283.0	234.0
	s_a^2		12,787	36,644			

(Continued)

Table 7. (Continued) Monthly Data (in Hours) for 1959 Delta Retreaded-Tire Service-Life by Retread Stage

Retread Stage		July	August	September	October	November	December
	N_a	33	35	21	28	25	18
1	$\bar{r}_a$	237.0	235.6	258.3	301.9	259.8	267.8
	s_a^2	5,449	4,016	2,908	2,110	4,471	1,858
	N_a	29	36	28	22	11	17
2	$\bar{r}_a$	228.8	242.7	230.8	261.2	258.0	322.9
	s_a^2	4,944	4,760	4,230	3,584	8,226	1,985
	N_a	14	15	20	20	30	16
3	$\bar{r}_a$	196.3	233.9	222.0	245.5	256.8	241.8
	s_a^2	6,110	6,474	3,444	1,787	5,149	7,088
	N_a	15	11	10	8	8	22
4	$\bar{r}_a$	211.1	199.0	267.4	215.9	265.3	257.8
	s_a^2	7,735	3,330	6,798	5,425	906	4,463
	N_a	4	7	5	2	7	3
5	$\bar{r}_a$	179.8	226.4	260.4	180.5	221.3	297.7
	s_a^2	5,662	5,403	2,700	1,513	10,307	6,631
	N_a	1	3	1	3	1	4
6	$\bar{r}_a$	185.0	188.7	190.0	230.7	255.0	243.5
	s_a^2		15,801		6,220		5,402

Table 8. Adjusted Monthly Landing Factors (MF_i) Used to Convert Delta Service-Life Data from Hours into Landings

ith Month	L_i Total Landings	H_i Total Flight Hours	LF_i Landings per Flight Hour	$MF_i = \frac{LF_{i-1} + LF_i}{2}$
4/57	7,291	6,922.97	1.053	1.053
5/57	7,802	7,121.30	1.096	1.075
6/57	7,501	6,825.15	1.099	1.098
7/57	7,592	6,753.12	1.124	1.112
8/57	7,764	6,935.30	1.119	1.121
9/57	7,180	6,630.93	1.083	1.101
10/57	7,454	6,554.67	1.137	1.110
11/57	6,844	6,039.02	1.133	1.135
12/57	7,138	6,188.62	1.153	1.143
1/58	7,430	6,381.08	1.164	1.158
2/58	6,885	5,989.02	1.150	1.157
3/58	7,525	6,604.50	1.139	1.145
4/58	7,505	6,487.03	1.157	1.148
5/58	7,242	6,171.67	1.173	1.165
6/58	6,969	6,020.40	1.158	1.165
7/58	7,073	6,075.62	1.164	1.161
8/58	7,026	6,092.85	1.153	1.158
9/58	7,129	6,309.53	1.130	1.142
10/58	7,356	6,426.25	1.145	1.138
11/58	6,811	6,068.03	1.122	1.134
1/59	7,296	6,733.60	1.084	1.086
2/59	6,547	5,882.97	1.113	1.099
3/59	7,804	6,819.28	1.144	1.128
4/59	7,558	6,602.62	1.145	1.145
5/59	7,478	6,744.47	1.109	1.127
6/59	7,204	6,476.85	1.112	1.111
7/59	7,494	6,780.78	1.105	1.109
8/59	7,725	7,033.70	1.098	1.101
9/59	7,298	6,639.00	1.099	1.099
10/59	7,538	6,930.03	1.088	1.094
11/59	7,018	6,389.33	1.098	1.093
12/59	7,370	6,603.30	1.116	1.107
1/60	7,222	6,633.05	1.089	1.102
2/60	7,373	6,312.60	1.168	1.129
3/60	7,581	6,519.53	1.163	1.166
4/60	7,979	6,538.97	1.220	1.192

where $n_a = N_a - 1$, $n = n_a$ and the s_a^2 are the independent estimates of variance to be compared, based respectively on n_a degrees of freedom. Bartlett has shown that if C is defined by the equation

$$C = 1 + \frac{1}{3(g-1)} \left[\sum_a \frac{1}{n_a} - \frac{1}{n} \right]$$

then the quantity M/C is somewhat more closely approximated by a χ^2 distribution with g-1 degrees of freedom than is M alone.

The results of this test and the critical $\chi^2_{.05}$ values at the five per cent level of significance are shown in Table 9. The actual M/C was greater than the critical $\chi^2_{.05}$ value in only one of the 12 months tested. Since the hypothesis was not rejected in 11 of the months tested, the hypothesis that the s_a^2 are homogeneous was not rejected on the basis of these tests.

The Braniff s_a^2 data in Table 10 were also used to test the hypothesis that the s_a^2 are homogeneous. Bartlett's test was used and the M/C value was found to be

$$M/C = 1.36 \qquad df = 5$$

The critical χ^2 value at the five per cent level of significance is

$$\chi^2_{.05} = 11.07$$

Since the actual chi-square value is less than the critical value, the hypothesis that the s_a^2 are homogeneous was not rejected.

On the basis of these analyses the following assumptions were made. Within months the s_a^2 for the several retread stages are homogeneous, and the s_r^2 of all retread stages combined can be used to estimate the s_a^2 of any retread stage.

Table 9. Results of Bartlett's Test of Hypothesis that Within Months the Service-Life Variances for the Retread Stages (s_a^2) are Homogeneous

Month	M/C Actual	$\chi^2_{.05}$	Month	M/C Actual	$\chi^2_{.05}$
January	10.19	9.49	July	1.01	11.07
February	9.55	11.07	August	3.81	11.07
March	10.95	11.07	September	2.82	9.49
April	5.46	9.49	October	5.60	11.07
May	8.29	9.49	November	9.34	9.49
June	4.02	9.49	December	9.96	11.07

Table 10. Braniff Service-Life Data (in Landings) from Removals Appearing in the "Tire Operation Summary" as Individual Removals for Period January 1957 Through October 1959

New Tires	N_n	$\bar{n}$	s_n	s_n^2
	88	292.9	107.2	11,490.4

(Continued)

Table 10. (Continued) Braniff Service-Life Data (in Landings) from Removals Appearing in the "Tire Operation Summary" as Individual Removals for Period January 1957 Through October 1959

Retread Stage	N_a	$\bar{r}_a$	s_a	s_a^2
1	172	349.1	129.9	16,881.9
2	147	343.3	129.0	16,642.8
3	140	321.4	132.0	17,426.7
4	128	316.0	121.9	14,863.8
5	57	312.3	124.3	15,446.1
6	33	308.9	118.4	14,022.1
All Retread Stages	N_r	$\bar{r}$	s_r	s_r^2
	677	330.8	128.1	16,414.9
All Removals	N_g	$\bar{g}$	s_g	s_g^2
	765	326.5	126.4	15,979.3

Effect of Retread Stage on Mean Service-Life of Retreaded Tires.--The hypothesis was established that the monthly $\bar{r}_a$ of each of the retread stages are equal to the $\bar{r}_i$ of all the retread stages combined. The Student's t-Test was used to test this hypothesis.

A two-tailed t-Test was conducted using the Delta monthly $\bar{r}_a$ data in Table 7, and the Delta monthly $\bar{r}_i$ and s_r^2 data in Table 11. The statistic used to test the hypothesis was (16)

$$t = \frac{\bar{r}_a - \bar{r}_i}{s_r / \sqrt{N_a}}$$

The utilization of s_r in lieu of s_a was substantiated by the analysis in the previous section which did not reject the hypothesis of the homogeneity of the monthly s_a^2. The results of these t-Tests are shown in Table 12. The hypothesis was rejected for the first and fourth retread stage in October, and the second retread stage in December. In the other 62 t-Tests, the hypothesis that the monthly $\bar{r}_a$ are equal to the monthly $\bar{r}_i$ was not rejected at the five per cent level of significance.

Table 11. Monthly 1959 Delta Data (in Hours) for New and Retreaded-Tire Service-Life Mean and Variance

Month	New Tires			Retreaded Tires		
	N_n	$\bar{n}$	s_n^2	N_r	$\bar{r}$	s_r^2
January	11	308.5	4,723.1	63	358.4	16,969.5
February	14	306.0	9,207.8	55	376.7	13,929.2
March	18	246.1	4,874.2	71	330.6	13,600.9
April	43	243.6	2,241.4	64	294.5	6,601.9
May	24	237.5	3,067.1	88	253.9	5,805.6
June	22	214.2	5,541.9	94	244.8	6,842.8
July	27	194.9	1,766.5	96	221.6	5,745.7
August	10	219.9	684.5	107	232.1	4,844.7
September	15	253.1	1,810.8	85	241.1	4,020.3
October	21	261.5	5,919.9	83	263.8	2,660.1
November	20	256.5	4,660.8	82	255.7	5,143.1
December	5	289.4	4,193.3	80	271.6	4,583.6

Table 12. Results of t-Tests on 1959 Monthly Delta Data to Test the Hypothesis that the Mean Service-Life for Each Retread Stage ($\bar{r}_a$) is Equal to the Mean Service-Life of All Retread Stages Combined ($\bar{r}_i$)

		Retread Stage					
Month		1	2	3	4	5	6
January	t	1.301	1.194	1.168	1.945	.315	
	$t_{.05}$	2.145	2.086	2.160	2.365	2.776	
February	t	1.606	1.363	.598	2.101	.396	.755
	$t_{.05}$	2.365	2.201	2.110	2.306	2.776	4.303
March	t	.167	.158	.321	.345	.639	1.498
	$t_{.05}$	2.131	2.093	2.160	2.228	2.571	3.182
April	t	.238	1.611	.411	.333	1.025	
	$t_{.05}$	2.069	2.201	2.120	2.365	4.303	
May	t	1.623	.760	.259	1.102	1.762	
	$t_{.05}$	2.040	2.145	2.086	2.145	3.182	
June	t	.184	.863	.386	1.648	1.809	
	$t_{.05}$	2.014	2.145	2.145	2.228	3.182	
July	t	1.167	.512	1.251	.540	1.105	
	$t_{.05}$	2.038	2.048	2.160	2.145	3.182	
August	t	.301	.915	.101	1.576	.214	1.080
	$t_{.05}$	2.034	2.032	2.145	2.228	2.447	4.303
September	t	1.246	.857	1.350	1.312	.681	
	$t_{.05}$	2.086	2.052	2.093	2.262	2.776	
October	t	3.899	.237	1.590	2.628	2.284	1.113
	$t_{.05}$	2.052	2.080	2.093	2.365	12.706	4.303
November	t	.287	.109	.901	.379	1.268	
	$t_{.05}$	2.064	2.228	2.045	2.365	2.447	
December	t	.234	3.129	1.758	.952	.784	.829
	$t_{.05}$	2.110	2.120	2.131	2.080	4.303	3.182

The hypothesis that the $\bar{r}_a$ are equal to the $\bar{r}$ was tested using time-independent data. The Braniff data in Table 12 and the Delta data in Table 13 were used with the statistic

$$t = \frac{\bar{r}_a - \bar{r}}{s_a / \sqrt{N_a}}$$

Table 13. Data (in Landings) from Random Sample of Delta Removals in Period April 1957 through September 1959

New Tires	N_n	$\bar{n}$	s_n	s_n^2
	405	270.5	85.5	7,311.83
Retread Stage	N_a	$\bar{r}_a$	s_a	s_a^2
1	330	317.3	96.4	9,292.1
2	282	308.9	104.4	10,902.7
3	187	303.5	108.7	11,807.8
4	108	318.0	117.7	13,849.9
5	47	310.6	125.3	15,706.1
6	13	301.3	125.3	15,701.6
All Retread Stages	N_r	$\bar{r}$	s_r	s_r^2
	967	311.7	105.9	11,224.8
All Removals	N_g	$\bar{g}$	s_g	s_g^2
	1,372	299.5	101.7	10,343.2

The results of these two-tailed t-Tests are shown in Tables 14 and 15. The hypothesis that the $\bar{r}_a$ are equal to the $\bar{r}$ was not rejected in any of the twelve tests at the five per cent level of significance.

The analysis of variance was used to test the hypothesis that the monthly $\bar{r}_a$ are equal and that month-to-month variability does not affect $\bar{r}_a$ (17). The Delta monthly $\bar{r}_a$ data in Table 7 were converted to landings by the MF_i in Table 8.

Table 14. Results of t-Tests on Delta Time-Independent Data to Test the Hypothesis that the Mean Service-Life for Each Retread Stage ($\bar{r}_a$) is Equal to the Mean Service-Life of All Retread Stages Combined ($\bar{r}$)

	Retread Stage					
	1	2	3	4	5	6
t	1.041	.452	1.033	.556	.062	.301
$t_{.05}$	1.960	1.960	1.970	1.990	2.015	2.179

Table 15. Results of t-Tests on Braniff Time-Independent Data to Test the Hypothesis that the Mean Service-Life for Each Retread Stage ($\bar{r}_a$) is Equal to the Mean Service-Life of All Retread Stages Combined ($\bar{r}$)

	Retread Stage					
	1	2	3	4	5	6
t	1.845	1.172	.847	1.372	1.131	1.054
$t_{.05}$	1.960	1.970	1.980	1.980	2.005	2.040

These data in landings are shown in Table 16. The rows in the analysis of variance consisted of the months of the year, and the columns consisted of the retread stages. The retread stages utilized were the first three stages, and the fourth, fifth, and sixth stages combined. The fourth, fifth, and sixth retread stages were combined to increase the mean service-life data.

Table 16. Mean Service-Life Data ($\bar{r}_a$ in Landings) for 1959 Monthly Delta Retread Stages

Month	Retread Stage			
	1	2	3	4 or more
January	436.7	352.3	345.0	441.4
February	487.7	465.1	395.7	362.7
March	378.3	377.5	384.2	356.8
April	332.6	380.5	327.9	314.1
May	310.8	269.2	291.0	254.3
June	272.2	251.5	281.2	281.8
July	262.8	253.7	217.7	225.7
August	259.4	267.2	257.5	227.5
September	283.9	253.6	244.0	286.2
October	330.3	285.8	268.6	234.8
November	284.0	282.0	280.7	268.2
December	296.5	357.5	267.7	287.8

The analysis of variance also was used with the monthly Braniff $\bar{r}_a$ data in Table 17 to test the hypothesis that the monthly $\bar{r}_a$ data for the several retread stages are equal and that month-to-month variability does not affect $\bar{r}_a$. The retread stages used were the same as for the

Delta data to provide larger sample sizes.

Table 17. Mean Service-Life Data ($\bar{r}_a$) for Monthly Braniff Retread Stages in Period November 1958 Through October 1959

Month	Retread Stage 1	2	3	4 or more
November, 1958	382.5	399.4	282.0	291.9
December, 1958	346.4	381.1	458.0	341.7
January, 1959	388.8	450.5	312.0	415.5
February, 1959	350.7	452.9	346.3	457.9
March, 1959	416.4	333.0	415.1	342.8
April, 1959	336.9	332.5	387.1	380.5
May, 1959	359.6	344.7	327.9	284.9
June, 1959	350.1	360.5	298.6	310.7
July, 1959	341.4	361.0	304.3	284.0
August, 1959	288.8	342.4	376.4	308.7
September, 1959	287.4	314.2	307.4	357.5
October, 1959	309.4	308.6	304.1	323.8

The results of these two analysis of variance tests are shown in Tables 18 and 19. The Delta data in Table 18 reject both the retread effect and monthly effect hypotheses. The Braniff data in Table 19 do not reject the retread effect hypothesis, but do reject the monthly effect hypothesis. All of these results are at the five per cent level of significance.

The above Delta data rejection of the hypothesis that the $\bar{r}_a$ for the several retread stages are equal, indicates that an increase in the retread stage may affect retreaded-tire service-life. However, the

Table 18. Analysis of Variance Table for 1959 Monthly Delta Mean Service-Life of Retread Stages

Source	Sum of Squares	Degrees of Freedom	Mean Square	Variance Ratio	$F_{.05}$
Retread Stage Effect	9,057.0	3	3,019.0	3.58	2.89
Monthly Effect	160,401.2	11	14,581.9	17.28	2.09
Residual	27,840.6	33	843.7		
Total	197,298.8				

Table 19. Analysis of Variance Table for Monthly Braniff Mean Service-Life of Retread Stages in Period November 1958 Through October 1959

Source	Sum of Squares	Degrees of Freedom	Mean Square	Variance Ratio	$F_{.05}$
Retread Stage Effect	4,211.9	3	1,404.0	.74	2.89
Monthly Effect	43,920.9	11	3,992.8	2.10	2.09
Residual	62,614.0	33	1,897.4		
Total	110,746.0				

results of the t-Tests and the Braniff data analysis of variance, led to the assumption that the retread stage does not significantly affect $\bar{r}_a$. It was previously assumed that the retread stage does not significantly affect s_a^2. These two assumptions led to the decision to combine all retread stages and utilize $\bar{r}$ and s_r^2 as estimates of retreaded-tire service-life mean and variance. In all of the subsequent analyses, the retread stage was not considered as a separate variable.

Comparison of New-Tire Service-Life Variance and Retreaded-Tire Service-Life Variance. --The next study was the comparison of s_n^2 and and s_r^2. This was necessary prior to the comparison of new-tire mean service-life ($\bar{n}$) and retreaded-tire mean service-life ($\bar{r}$).

The hypothesis that the 1959 monthly Delta s_r^2 in Table 20 are homogeneous among months was tested by Bartlett's test. The data in Table 20 were obtained by applying the monthly MF_i in Table 8 to the hourly s_r^2 data in Table 11. The value of M/C was found to be

$$M/C = 130.4$$

The hypothesis that the 1959 monthly Delta s_n^2 data in Table 20 are homogeneous among months was also tested by Bartlett's test. The value of M/C was found to be

$$M/C = 32.3$$

The critical chi-square value for these two tests is

Table 20. Monthly 1959 Delta Data (in Landings) for New and Retreaded-Tire Service-Life Mean and Variance ($\bar{n}_i$, $\bar{r}_i$, s_n^2, s_r^2)

Month	N_n	$\bar{n}_i$	s_n	s_n^2	N_r	$\bar{r}_i$	s_r	s_r^2
January	11	335.0	74.6	5,570.4	63	389.2	141.5	20,013.9
February	14	336.3	105.5	11,121.2	55	414.0	129.7	16,823.7
March	18	277.6	78.8	6,201.9	71	372.9	131.5	17,305.8
April	43	278.9	54.2	2,938.7	64	337.2	93.0	8,655.8
May	24	267.6	62.4	3,895.6	88	290.7	85.9	7,373.7
June	22	238.0	82.7	6,839.4	94	272.0	91.9	8,446.0
July	27	216.1	46.6	2,172.6	96	245.8	84.1	7,066.6
August	10	242.1	28.8	829.8	107	255.5	76.6	5,872.7
September	15	278.1	46.8	2,187.1	85	265.0	69.7	4,855.7
October	21	286.1	84.2	7,084.9	83	288.6	56.4	3,183.6
November	20	280.4	74.6	5,567.8	82	279.4	78.4	6,144.0
December	5	320.4	71.7	5,138.5	80	300.6	74.9	5,616.8

$$\chi^2_{.05} = 19.7$$

Since the M/C value is greater than the critical $\chi^2_{.05}$ value for both tests, the hypotheses that the monthly s_n^2 and s_r^2 are homogeneous were rejected. On the basis of these results neither the monthly s_n^2 nor s_r^2 were assumed to be homogeneous.

Since the s_n^2 and s_r^2 are not homogeneous, it was necessary to test the hypothesis that s_n^2 equals s_r^2 on a within month basis. The F-Test was used to test this hypothesis with the monthly Delta s_n^2 and s_r^2 in Table 20 (18). The results of these F-Tests are shown in Table 21. In seven of the twelve months tested, s_r^2 was greater than s_n^2 at the five per cent level of significance. In four of the months tested, s_r^2 was greater than s_n^2, but was not greater at the five per cent level of significance. In only one of the months tested was s_n^2 greater than s_r^2 at the five per cent level of significance.

The hypothesis that $s_n^2 = s_r^2$ was also tested with time-independent Delta data in Table 13 and the time-independent Braniff data in Table 10. The actual value of F for the Delta data was found to be

$$F = 1.54$$

The critical F value for the Delta data is

$$F_{.05} = 1.19$$

The actual value of F for the Braniff data was found to be

Table 21. Results of F-Tests on Monthly 1959 Delta New-Tire Service-Life Variance (s_n^2) and Retreaded-Tire Service-Life Variance (s_r^2) to Test the Hypothesis that $s_n^2 = s_r^2$

Month	$F = s_r^2 / s_n^2$	df_1	df_2	$F_{.05}$
January	3.59	62	10	2.62
February	1.51	54	13	2.31
March	2.79	70	17	2.05
April	2.95	63	42	1.62
May	1.89	87	23	1.84
June	1.23	93	21	1.88
July	3.25	95	26	1.76
August	7.08	106	9	2.76
September	2.22	84	14	2.20
October	.22	82	20	.52
November	1.10	81	19	1.96
December	1.09	79	4	5.67

$$F = 1.43$$

The critical F value for the Braniff data is

$$F_{.05} = 1.33$$

In both of these tests the actual F value was greater than the $F_{.05}$ value. Therefore, the hypothesis that $s_n^2 = s_r^2$ is rejected on the basis of these Delta and Braniff time-independent data.

On the basis of the F-Test results for the within month and time-independent data, it was assumed that s_n^2 was not equal to s_r^2.

<u>Comparison of New-Tire Mean Service-Life and Retreaded-Tire Mean Service-Life</u>. --The hypothesis was established that the monthly mean service-life of retreaded tires ($\bar{r}_i$) is equal to or less than the monthly mean service-life of new tires ($\bar{n}_i$). It was previously shown that s_n^2 and s_r^2 are not equal; therefore, the Aspin-Welch test was utilized (19). The Aspin-Welch test consists in treating

$$t = \frac{\bar{r}_i - \bar{n}_i}{\sqrt{s_n^2/N_n + s_r^2/N_r}}$$

as if it had a t-distribution with degrees of freedom given by

$$df = \frac{1}{c^2/N_r - 1 + (1-c)^2/N_n - 1}$$

where

$$c = \frac{s_r^2}{s_r^2/N_r + s_n^2/N_n}$$

These tests were conducted with the 1959 monthly Delta data in Table 20. The results of these tests are shown in Table 22. At the five per cent level of significance the hypothesis was rejected in six of the months tested, but was not rejected in the other six months.

There is a partial explanation for two of the three months in which

Table 22. Results of Aspin-Welch Tests on Monthly 1959 Delta New-Tire Mean Service-Life ($\bar{n}_i$) and Retreaded-Tire Mean Service-Life ($\bar{r}_i$) to Test the Hypothesis that $\bar{r}_i$ is Less than or Equal to $\bar{n}_i$

Month	t	df	$t_{.05}$
January	1.889	26	1.706
February	2.342	24	1.711
March	3.923	45	1.681
April	4.075	100	1.668
May	1.470	50	1.677
June	1.703	33	1.692
July	2.395	77	1.666
August	1.144	24	1.711
September	-.927	26	1.706
October	.131	25	1.708
November	-.049	30	1.697
December	-.597	5	2.015

the $\bar{n}_i$ was greater than the $\bar{r}_i$. A representative of the Gordy Tire Company, Atlanta, Georgia, gave the following information relative to the November and December 1959 $\bar{r}_i$ values. The Gordy Tire Company was performing all of the retreading for Delta in October 1959. Many of the November and December retreaded-tire removals were retreaded in October, when the Gordy Tire Company was using a tread rubber compound which yielded a much lower service-life than compounds used previously and subsequently. This compound was used on approximately 65 tires in October and was considered by the Gordy Tire Company representative to be responsible for $\bar{r}_i$ being less than $\bar{n}_i$ in November and

December.

The hypothesis that $\bar{r}_i$ was equal to or less than $\bar{n}_i$ was also tested with the Braniff $\bar{n}_i$ and $\bar{r}_i$ data in Table 23. Monthly s_n^2 and s_r^2 were not available for the Braniff data because of the characteristics of the Monthly Tire Operation Summary. Therefore, it was assumed that the time-independent s_n^2 and s_r^2 in Table 10 were equal to the monthly s_n^2 and s_r^2 for the period January 1957 through October 1959. These data were tested with a one-tail test using the statistic (20)

$$Z = \frac{\bar{r}_i - \bar{n}_i}{\sqrt{s_n^2/N_n + s_r^2/N_r}}$$

The critical value of Z at the five per cent level of significance is

$$Z_{.05} = 1.645$$

The hypothesis was rejected when the actual Z was greater than the $Z_{.05}$. The results of these tests are shown in Table 24. In eight of the 34 tests the hypothesis was rejected at the five per cent level of significance. The other 26 tests did not reject the hypothesis that $\bar{r}_i$ is less than or equal to $\bar{n}_i$ at the five per cent level of significance.

The preceding Aspin-Welch tests and Z tests did not provide any conclusive evidence as to whether the hypothesis that $\bar{r}_i$ is equal to or less than $\bar{n}_i$ should be accepted or rejected. The disadvantage in using

Table 23. Monthly Braniff New-Tire Mean Service-Life ($\bar{n}_i$) and Retreaded-Tire Mean Service-Life ($\bar{r}_i$) for the Period January 1957 Through October 1959

	1957				1958			
	N_n	$\bar{n}_i$	N_r	$\bar{r}_i$	N_n	$\bar{n}_i$	N_r	$\bar{r}_i$
January	12	343.9	42	398.2	14	405.5	60	421.9
February	14	357.7	38	434.8	13	363.2	53	397.8
March	21	361.8	51	339.1	10	434.0	52	421.2
April	18	363.8	52	363.1	13	356.5	48	400.5
May	20	299.4	60	347.6	19	364.6	59	369.1
June	22	360.3	56	317.9	13	245.9	82	312.8
July	11	268.9	71	317.3	25	269.4	74	312.4
August	34	325.6	52	297.6	24	304.3	59	298.2
September	32	303.0	49	338.1	22	290.2	45	302.6
October	29	309.3	41	374.4	22	228.6	74	342.9
November	26	397.8	42	391.3	44	332.6	37	348.6
December	25	397.6	30	401.7	49	342.9	36	359.3

	1959				Combined Months for Period January 1957 Through October 1959			
	N_n	$\bar{n}_i$	N_r	$\bar{r}_i$	N_n	$\bar{n}_i$	N_r	$\bar{r}_i$
January	21	366.2	34	415.5	47	372.2	136	413.0
February	31	333.0	53	390.9	58	345.7	144	405.0
March	13	404.5	43	395.7	44	390.8	146	385.0
April	16	336.1	82	346.9	47	352.3	182	365.7
May	7	335.4	104	345.0	46	331.8	223	352.0
June	14	278.4	73	344.0	49	306.6	211	325.0
July	31	319.6	57	336.8	67	292.6	202	321.0
August	35	322.7	70	340.7	93	319.0	181	314.5
September	30	280.3	97	312.5	84	291.5	191	316.8
October	7	320.3	74	312.4	58	280.0	189	337.8
November					70	356.8	79	371.3
December					74	361.4	66	378.6

Table 24. Results of Z-Tests on Monthly Braniff New-Tire Mean Service-Life ($\bar{n}_i$) and Retreaded-Tire Mean Service-Life ($\bar{r}_i$) to Test the Hypothesis that $\bar{r}_i$ is Less than or Equal to $\bar{n}_i$

Month	Value of Z	Month	Value of Z
January, 1957	1.480	June, 1958	2.032
February, 1957	2.177	July, 1958	1.648
March, 1957	-.801	August, 1958	-.220
April, 1957	-.002	September, 1958	.415
May, 1957	1.656	October, 1958	4.187
June, 1957	-1.481	November, 1958	.627
July, 1957	1.354	December, 1958	.624
August, 1957	-1.093	January, 1959	1.536
September, 1957	1.331	February, 1959	2.218
October, 1957	2.307	March, 1959	-.245
November, 1957	-.225	April, 1959	.355
December, 1957	.128	May, 1959	.225
January, 1958	.495	June, 1959	2.031
February, 1958	1.001	July, 1959	.670
March, 1958	-.334	August, 1959	.761
April, 1958	1.258	September, 1959	1.371
May, 1958	.152	October, 1959	.182

the Aspin-Welch tests is evident. Even if the population means were the same for every $\bar{n}_i$ and $\bar{r}_i$, it would be expected that if there were enough sample means some would be extremely large and some extremely small.

Six of the 12 Aspin-Welch tests of Delta data rejected the hypothesis at the five per cent level of significance. Eight of the Z tests of the Braniff data rejected the hypothesis at the five per cent level of significance. The question that had to be answered was which (if any) of these significant differences are really indicative of true differences between $\bar{n}_i$ and $\bar{r}_i$. The procedure used to answer this question was the analysis of variance (21).

The analysis of variance was applied to the monthly 1959 Delta $\bar{n}_i$ and $\bar{r}_i$ data in Table 20. The results are shown in Table 25.

The analysis of variance also was applied to the monthly Braniff $\bar{n}_i$ and $\bar{r}_i$. The Braniff data used were the combined monthly January 1957 through October 1959 data in Table 23. The results are shown in Table 26.

These two analysis of variance tests each tested the following hypotheses:

(1) There is no difference between $\bar{n}_i$ and $\bar{r}_i$.

(2) Monthly variability does not affect $\bar{n}_i$ and $\bar{r}_i$.

The analysis of variance results for the Delta and Braniff data rejected the above two hypotheses at the five per cent level of significance.

These results led to the conclusion that the monthly $\bar{r}_i$ is greater than the monthly $\bar{n}_i$ for both Delta and Braniff. The conclusion was also made that the month-to-month variation in $\bar{n}_i$ and $\bar{r}_i$ is significant.

<u>Three-Way Analysis of Variance to Determine Effect of Seasons, Tire Manufacturers, and Tire Retreading</u>.--The Braniff $\bar{n}_i$ and $\bar{r}_i$ monthly data in Table 23 was further classified by the manufacturer producing the new-tire or the retreaded-tire carcass. Tires which were retreaded by independent retreaders were classified by the manufacturer producing the retreaded-tire carcass. Table 27 shows this data with the $\bar{n}_i$

Table 25. Analysis of Variance Table for Monthly 1959 Delta $\bar{n}_i$ and $\bar{r}_i$ Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	Variance Ratio	$F_{.05}$
Tire $\bar{n}_i$ and $\bar{r}_i$ effect	5,230.3	1	5,230.3	8.04	4.84
Monthly effect	42,346.6	11	3,849.7	5.92	2.82
Residual	7,158.7	11	650.8		
Total	54,735.6				

Table 26. Analysis of Variance Table for Braniff Combined Monthly $\bar{n}_i$ and $\bar{r}_i$ Data for Period January 1957 Through October 1959

Source	Sum of Squares	Degrees of Freedom	Mean Square	Variance Ratio	$F_{.05}$
Tire $\bar{n}_i$ and $\bar{r}_i$ effect	3,384.4	1	3,384.4	15.89	4.84
Monthly effect	24,603.6	11	2,236.7	10.50	2.82
Residual	2,343.0	11			
Total	30,331.0				

Table 27. Braniff Monthly New-Tire Mean Service-Life ($\bar{n}_i$) and Retreaded-Tire Mean Service-Life ($\bar{r}_i$) Classified by Manufacturers and Seasons

		Manufacturer					
		a		b		c	
	Month	$\bar{n}_i$	$\bar{r}_i$	$\bar{n}_i$	$\bar{r}_i$	$\bar{n}_i$	$\bar{r}_i$
	December	350.0	400.9	414.6	412.4	319.8	325.8
Season	January	411.0	398.2	399.0	410.3	359.7	402.1
I	February	533.8	388.2	302.4	441.1	334.7	345.2
	March	409.0	406.5	413.4	372.8	332.8	424.5
	April	354.0	375.5	345.1	384.1	402.0	346.5
Season	May	303.4	355.2	352.5	319.8	288.6	361.5
II	October	289.0	308.7	274.2	336.5	147.0	290.7
	November	353.5	417.9	379.8	349.1	371.0	455.0
	June	362.6	340.0	306.8	318.5	327.3	339.7
Season	July	311.0	322.8	314.8	300.5	226.5	328.0
III	August	313.9	314.1	327.5	296.3	283.0	258.3
	September	293.7	304.4	313.5	307.5	254.0	316.5

		Manufacturer			
		d		e	
	Month	$\bar{n}_i$	$\bar{r}_i$	$\bar{n}_i$	$\bar{r}_i$
	December	340.6	379.5	229.0	343.6
Season	January	367.8	415.1	335.0	435.1
I	February	332.1	401.0	428.0	378.3
	March	381.5	365.5	439.0	426.7
	April	344.8	365.8	383.0	353.3
Season	May	293.6	362.6	378.0	410.1
II	October	293.7	365.0	317.3	361.9
	November	315.0	390.1	275.0	291.5
	June	271.1	325.8	223.0	312.1
Season	July	244.2	322.1	320.3	388.0
III	August	328.8	342.8	255.0	358.2
	September	258.8	324.7	299.2	333.9

and $\bar{r}_i$ data combined for identical months in the period January 1957 through October 1959. These data were combined in order to increase the cell sample sizes. The 12 monthly sets of $\bar{n}_i$, $\bar{r}_i$ and manufacturer data were classified into three seasons on the basis of similarities in monthly climatological temperatures (see Chapter II, Basis for Dividing the Calendar Year into Three Seasons).

A three-way classification analysis of variance was applied to the mean service-life data in Table 27 (22). Each cell value, X_{ijkl} is the outcome of the lth experiment using the ith treatment of factor A, and jth treatment of factor B and the kth treatment of factor C, where

1. Factor A was the five manufacturers (a, b, c, d, and e).
2. Factor B was the three seasons (I, II, and III).
3. Factor C was the tire level (new or retreaded).

Table 28 presents the results of the analysis of variance. The hypotheses tested are as follows:

1. The tire level (new or retreaded) does not affect mean service-life.
2. Seasonal variability does not affect mean service-life.
3. The different manufacturers do not affect mean service-life.

Interaction between ABC, AB, AC, and BC was not found to be significant at the five per cent level. The tire level and seasonal effects were

Table 28. Analysis of Variance Table for Braniff Monthly $\bar{n}_i$ and $\bar{r}_i$ Data Classified by Manufacturers and Seasons

Source	Sum of Squares	Degrees of Freedom	Mean Square	Variance Ratio	$F_{.05}$
Manufactuers effect (A)	14,069.5	4	3,517.4	1.68	2.47
Seasonal effect (B)	114,073.2	2	57,036.6	27.23	3.10
Tire level effect (C)	26,973.0	1	26,973.0	12.88	3.95
Interaction A and B	5,790.0	8	723.8	.35	2.04
Interaction A and C	11,350.1	4	2,837.5	1.35	2.47
Interaction B and C	1,144.5	2	572.3	.27	3.10
Interaction A, B and C	9,500.2	8	1,187.5	.57	2.04
Within cells	188,505.9	90	2,094.5		
Total	371,406.4	119			

significant at the five per cent level. The manufacturer effect was not significant at the five per cent level.

These results are in agreement with results found in the two-way analysis of variance tests with monthly $\bar{n}_i$ and $\bar{r}_i$ service-life data. In subsequent analyses the seasonal variation and tire level effects will be studied without respect to the manufacturer.

Correlation of Mean Tire Service-Life and Monthly Temperatures. -- Figure 3 presents the Braniff monthly new-tire mean service-life ($\bar{n}_i$) and retreaded-tire mean service-life ($\bar{r}_i$) shown in Table 23. Also shown in Figure 3 are the monthly adjusted Atlanta mean temperatures (t_i) shown in Table 30, which are based on the q_i temperatures shown in Table 29. The q_i temperatures in Table 29 are the monthly mean temperatures for Atlanta, Georgia, as furnished by the United States Department of Commerce. The t_i in Table 30 for the ith month were obtained by

$$t_i = \frac{q_i + q_{i-1}}{2}$$

Figure 4 presents the 1959 Delta $\bar{n}_i$ and $\bar{r}_i$ data in Table 20 plotted with t_i. Figure 4 was based on only 12 monthly points, since $\bar{n}_i$ and $\bar{r}_i$ monthly Delta data were only obtained for the year 1959.

Figure 5 presents the monthly mean service-life data for new and retreaded tires combined ($\bar{g}_i$), plotted with t_i. These $\bar{g}_i$ data for

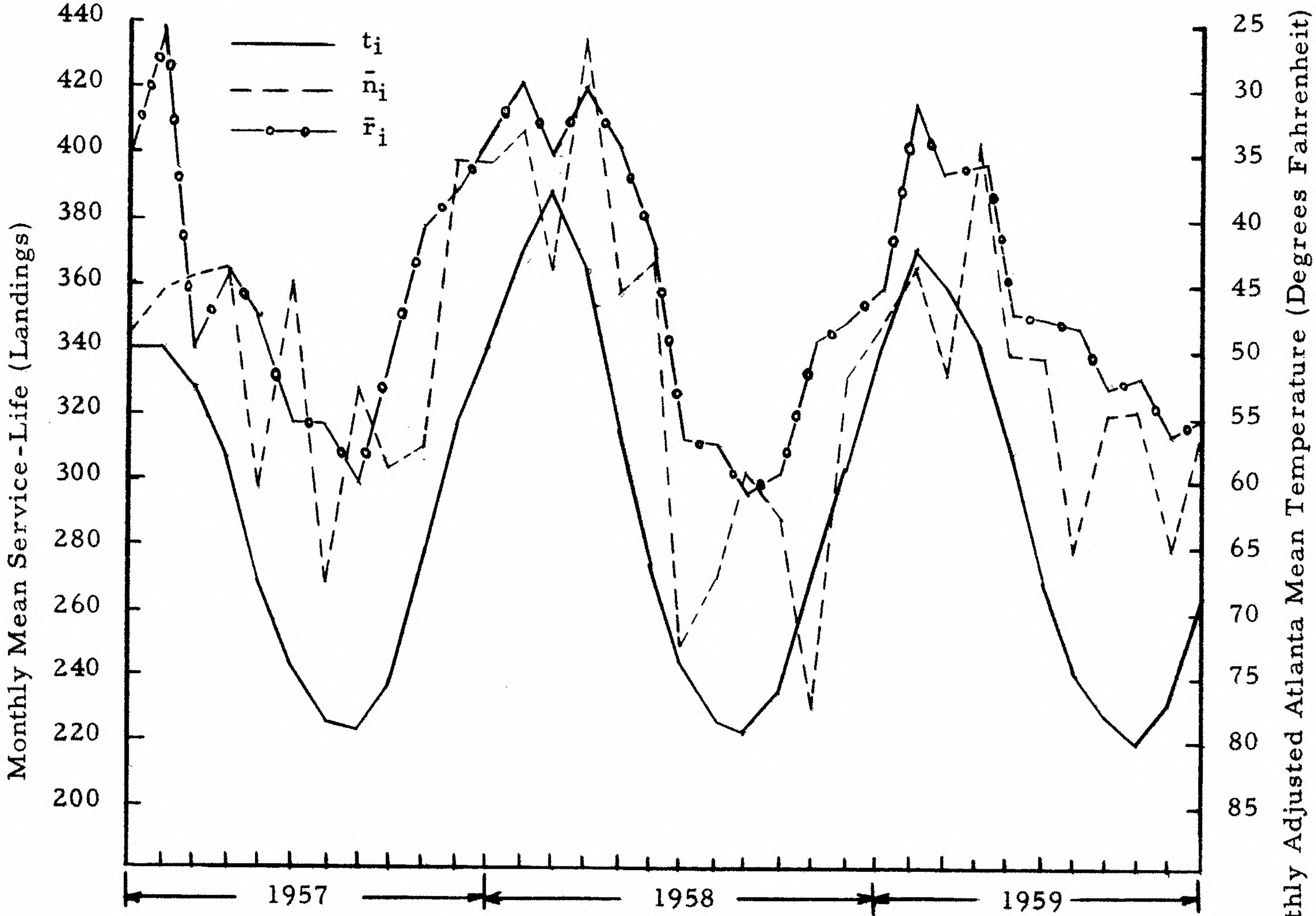


Figure 3. Variation of Braniff Monthly New-Tire Mean Service-Life ($\bar{n}_i$), Braniff Monthly Retreaded-Tire Mean Service-Life ($\bar{r}_i$), and Monthly Adjusted Atlanta Mean Temperature (t_i)

Table 29. Monthly Mean Temperatures (q_i) in Atlanta, Georgia

ith Month	1956	1957	1958	1959	1960
January		45. 9	38. 9	42. 2	44. 2
February		54. 7	37. 8	47. 7	43. 8
March		51. 5	49. 8	51. 2	41. 8
April		64. 3	62. 4	63. 0	63. 7
May		70. 8	70. 7	72. 2	
June		77. 8	77. 0	76. 6	
July		79. 0	78. 8	79. 1	
August		79. 5	78. 9	80. 6	
September		73. 3	74. 0	73. 3	
October		58. 4	62. 0	64. 8	
November		53. 7	56. 5	51. 8	
December	54. 3	46. 7	42. 7	46. 3	

Table 30. Monthly Adjusted Mean Temperatures (t_i) in Atlanta, Georgia

ith Month	1957	1958	1959	1960
January	50. 10	42. 80	42. 45	45. 25
February	50. 30	38. 35	44. 95	44. 00
March	53. 10	43. 80	49. 45	42. 80
April	57. 90	56. 10	57. 10	52. 75
May	67. 55	66. 55	67. 60	
June	74. 30	73. 85	74. 40	
July	78. 40	77. 90	77. 85	
August	79. 25	78. 85	79. 85	
September	76. 40	76. 45	76. 95	
October	65. 85	68. 00	69. 05	
November	56. 05	59. 25	58. 30	
December	50. 20	49. 60	49. 05	

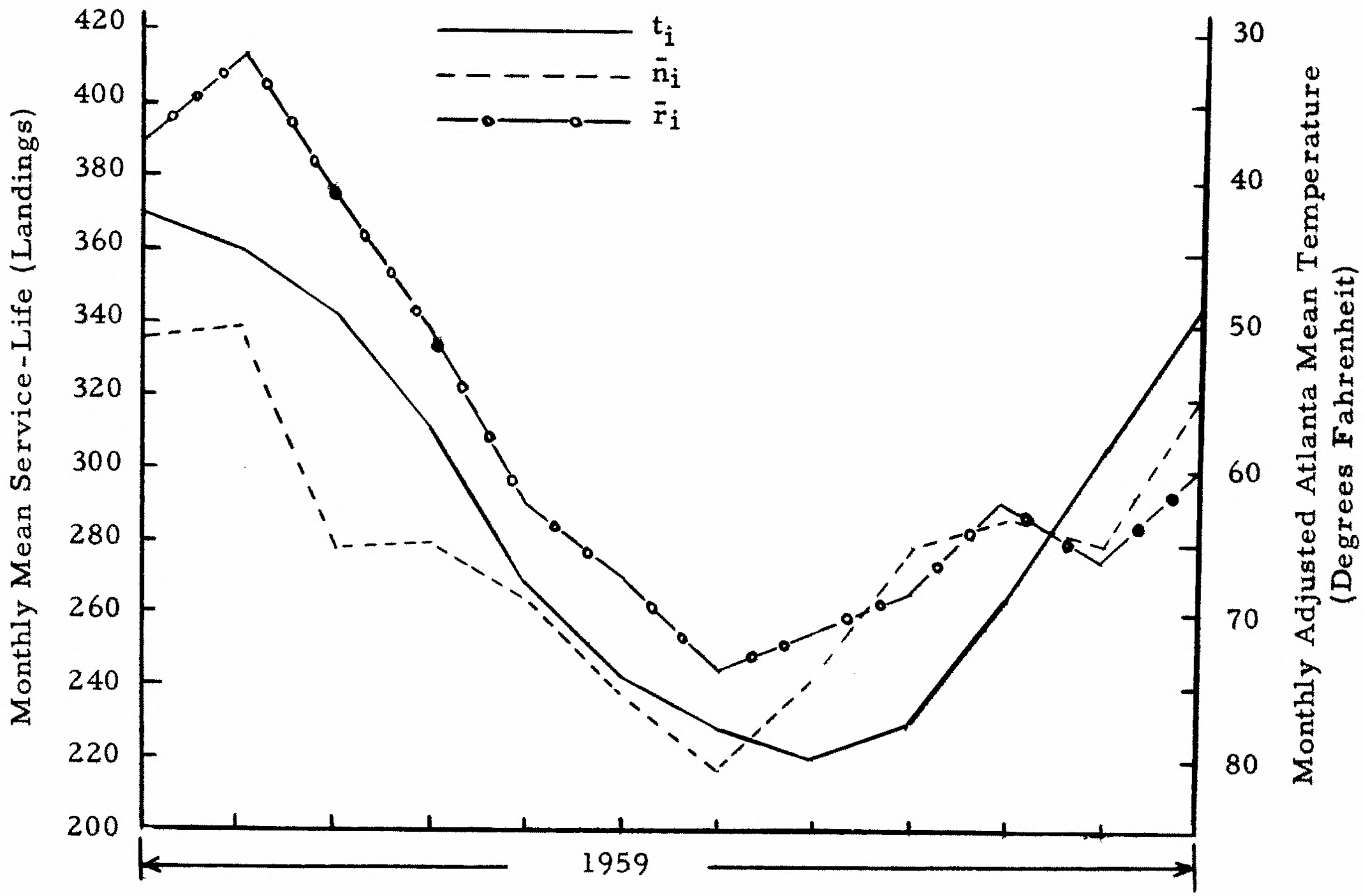


Figure 4. Variation of Delta Monthly New-Tire Mean Service-Life ($\bar{n}_i$) Delta Monthly Retreaded-Tire Mean Service-Life ($\bar{r}_i$), and Monthly Adjusted Atlanta Mean Temperature (t_i)

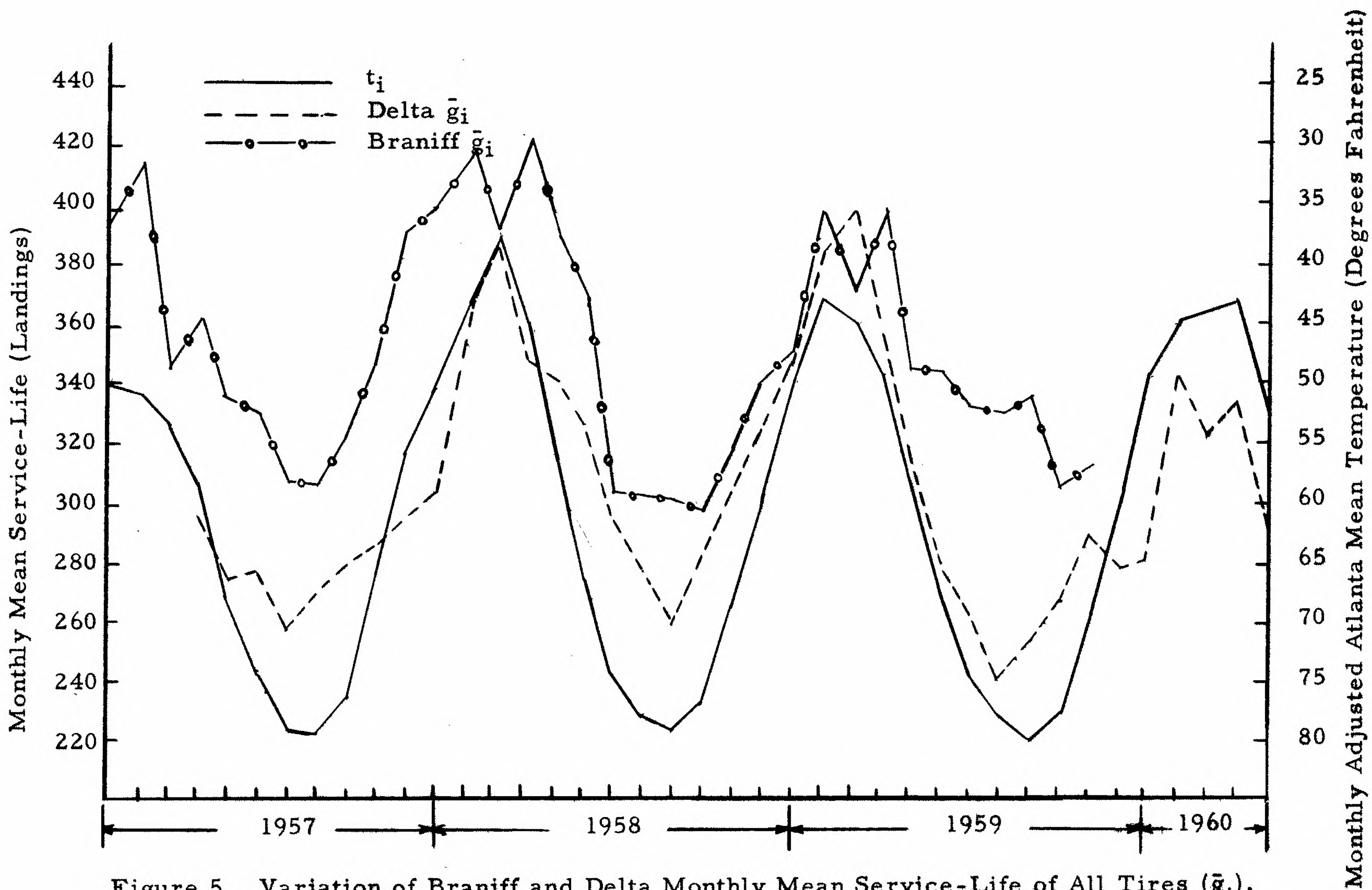


Figure 5. Variation of Braniff and Delta Monthly Mean Service-Life of All Tires ($\bar{g}_i$), and Monthly Adjusted Atlanta Mean Temperature (t_i)

Delta and Braniff are shown in Tables 31 and 32 respectively. The Delta data in Table 31 were obtained by applying the landing factor (MF_i) in Table 8 to the hourly data in Appendices B and C. The Braniff data in Table 32 were obtained by combining the $\bar{n}_i$ and $\bar{r}_i$ data in Table 23.

Figures 3 through 5 all show a definite seasonal fluctuation in $\bar{n}_i$, $\bar{r}_i$ and $\bar{g}_i$ which is inversely related to t_i. Regression lines using the method of least squares were fitted to the monthly service-life means and t_i (23). The least squares equation was of the form

$$y = b_o + b_i x$$

where y = estimate of $\bar{n}_i$, $\bar{r}_i$ or $\bar{g}_i$

x = t_i temperature in month of estimate

The regression line equation, $s_{y/x}$, and the sample correlation coefficient r were calculated for the data shown in Figures 3 through 5. A measure of the degree of association between the mean monthly service-life and monthly t_i is the correlation coefficient ρ. An estimate of ρ is given by the sample correlation coefficient r. A significance test for $\rho = 0$ is equivalent to testing whether or not a linear relation is present (24). To test the hypothesis that $\rho = 0$, reject when

$$t = \frac{r}{\sqrt{1 - r^2}} \sqrt{n - 2} \geq t_{.05}$$

Table 31. Delta Monthly Mean Service-Life (in Landings) for New and Retreaded Tires Combined ($\bar{g}_i$) for Period April 1957 Through April 1960

Month	1957	1958	1959	1960
January		365.7	385.4	342.8
February		388.8	397.6	321.5
March		349.6	353.6	332.9
April	297.2	339.7	313.7	293.5
May	274.1	327.0	282.2	
June	276.1	294.9	265.5	
July	256.8	276.8	240.6	
August	270.7	261.0	253.1	
September	280.7	285.8	267.5	
October	284.4	305.3	288.1	
November	294.4	324.0	279.6	
December	304.5	346.1	302.8	

Table 32. Braniff Monthly Mean Service-Life for New and Retreaded Tires Combined ($\bar{g}_i$) for Period January 1957 Through October 1959

Month	1957	1958	1959
January	386.2	418.8	396.7
February	414.0	391.0	369.5
March	345.7	423.3	397.8
April	363.3	391.1	345.1
May	335.5	368.0	344.4
June	329.9	303.6	333.5
July	310.8	301.6	330.8
August	308.6	299.9	334.7
September	324.3	298.5	304.9
October	347.4	316.7	313.1
November	393.8	339.9	
December	399.8	349.9	

Table 33 presents the regression line equation, $s_{y/x}$, sample correlation coefficient r, and the results of the tests of the hypothesis that $\rho = 0$.

Table 33 presents least squares correlation results for Delta $\bar{r}_i$ for the full 12 months in 1959, and for the ten months of January through October 1959. The latter correlation was conducted because of the poor service-life from November and December retreaded tires which was attributed to the inferior tire-tread compound used. The correlation without November and December data yielded an increased correlation coefficient r, and a decreased $s_{y/x}$.

The results in Table 33 show that the hypothesis that $\rho = 0$ was rejected for all seven sets of data at the five per cent level of significance. The seven regression equations in Table 33 are presented in Figure 6. The slopes of all the regression lines, excluding the Delta $\bar{r}_i$, are approximately the same. The greater slope of the two Delta $\bar{r}_i$ regression lines may have been due to sampling errors resulting from the smaller sample sizes of ten and twelve months. Excluding the Delta $\bar{r}$ regression lines, the observation that the regression lines are nearly parallel (i. e., the slopes are nearly equal) substantiates the conclusion that the temperature t_i has a consistent relationship to $\bar{n}$, $\bar{r}$ and $\bar{g}$.

The regression lines in Figure 6 show that for a given t_i the Braniff $\bar{n}_i$, $\bar{r}_i$ and $\bar{g}_i$ are greater than the respective $\bar{n}_i$, $\bar{r}_i$ and $\bar{g}_i$ of

Table 33. Regression Line Equations, $s_{y/x}$, Correlation Coefficients r, and Results of the Tests of the Hypothesis that $\rho = 0$ for the Monthly Mean Service-Life Data and t_i Shown in Figures 3 Through 5

	Number of Months	Regression Line Equation	$s_{y/x}$	r	t	$t_{.05}$
Braniff $\bar{n}_i$	34	$\bar{n}_i = 493.7 - 2.57t_i$	33.5	-.718	5.80	2.04
Braniff $\bar{r}_i$	34	$\bar{r}_i = 520.7 - 2.61t_i$	20.3	-.866	9.79	2.04
Braniff $\bar{g}_i$	34	$\bar{g}_i = 508.7 - 2.52t_i$	19.0	-.871	9.85	2.04
Delta $\bar{n}_i$	12	$\bar{n}_i = 424.2 - 2.32t_i$	20.5	-.852	5.18	2.23
Delta $\bar{r}_i$	12	$\bar{r}_i = 532.5 - 3.59t_i$	27.7	-.882	5.94	2.23
Delta $\bar{r}_i$	10	$\bar{r}_i = 580.1 - 4.18t_i$	10.9	-.985	16.37	2.31
Delta $\bar{g}_i$	37	$\bar{g}_i = 459.7 - 2.51t_i$	21.1	-.855	9.73	2.03

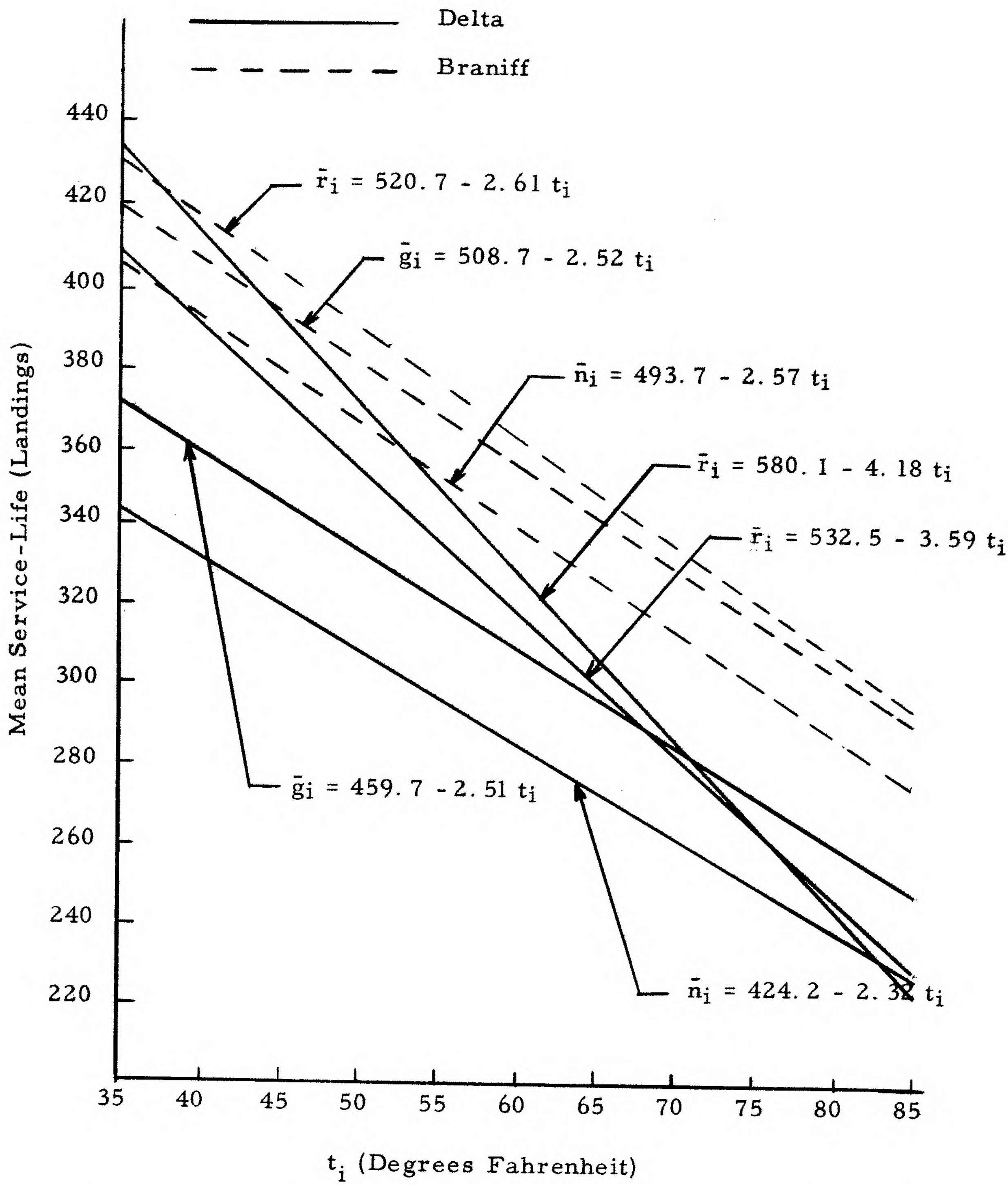


Figure 6. Least Squares Regression Equations for Braniff and Delta Monthly Mean Service-Life ($\bar{n}_i$, $\bar{r}_i$, $\bar{g}_i$) and Monthly Adjusted Atlanta Mean Temperature (t_i)

Delta. The consistently higher Braniff $\bar{n}_i$, $\bar{r}_i$ and $\bar{g}_i$ may be partially explainable. A representative of the Gordy Tire Company, which performs retreading for both Braniff and Delta, stated that the unserviceable tires from Braniff appeared to be worn to lower limits than the unserviceable tires from Delta. No investigation was conducted of any other possible causes for the differences among the Braniff and Delta mean service-life data. It was assumed that differences in management policy and aircraft operating characteristics were the causes of the differences, and not the variable t_i.

The effect of retreaded tire utilization on the mean service-life of all tires ($\bar{g}$) was next studied. The Braniff $\bar{n}_i$ and $\bar{r}_i$ regression equations in Table 33 were combined to yield the following equation to estimate $\bar{g}_i$.

$$\bar{g}_i = 493.7 - 2.57\ t_i + 26.92\ K_r - .04\ t_i\ K_r$$

The Delta $\bar{n}_i$ and $\bar{r}_i$ (for January through October 1959) regression equations were combined to yield the following equation to estimate $\bar{g}_i$.

$$\bar{g}_i = 424.2 - 2.32\ t_i + 155.9\ K_r - 1.85\ t_i\ K_r$$

In the above equations K_r represents the proportion of tires used that are retreaded tires. These two equations were obtained by multiplying the $\bar{n}_i$ and $\bar{r}_i$ regression equations by $(1 - K_r)$ and K_r, respectively, and totaling the results.

Figure 7 presents these $\bar{g}$ equation lines for the range of t_i considered (35 to 85 degrees), and several values of K_r. A K_r of .750 means that three retreaded tires are used for every new tire. Figure 7 shows the effect of the greater slope of the Delta $\bar{r}$ regression line equation shown in Table 33 upon the Delta $\bar{g}$ equation lines. As t_i becomes greater, the value of K_r has a lesser effect on Delta $\bar{g}$. The Braniff $\bar{g}$ equation lines in Figure 7 have nearly the same slope because the $\bar{n}$ and $\bar{r}$ regression line equations shown in Table 33 have similar slopes. It is to be noted that t_i varied from 38.35 to 79.85 degrees in the period January 1957 through October 1959.

The $\bar{g}$ equation lines in Figure 7 provide a method of forecasting $\bar{g}$, since forecasts can be made of K_r and t_i. The next analysis will show the relationship of $\bar{g}$ to the inventory demand parameter, landings per assembly removal.

Correlation of the Mean Service-Life of All Assembly Removals and Landings per Assembly Removal. --The previous analyses have shown that the monthly mean service-life of new and retreaded tires combined ($\bar{g}$) is related to t_i temperatures. The inventory demand parameter was previously stated to be the quantity, landings per assembly removal (L/R). It was next desirable to study the relationship between $\bar{g}$ and L/R.

Table 34 shows monthly Delta L_i/R_i and Table 31 shows monthly Delta $\bar{g}_i$. Since the aircraft studied has four wheel assemblies, the

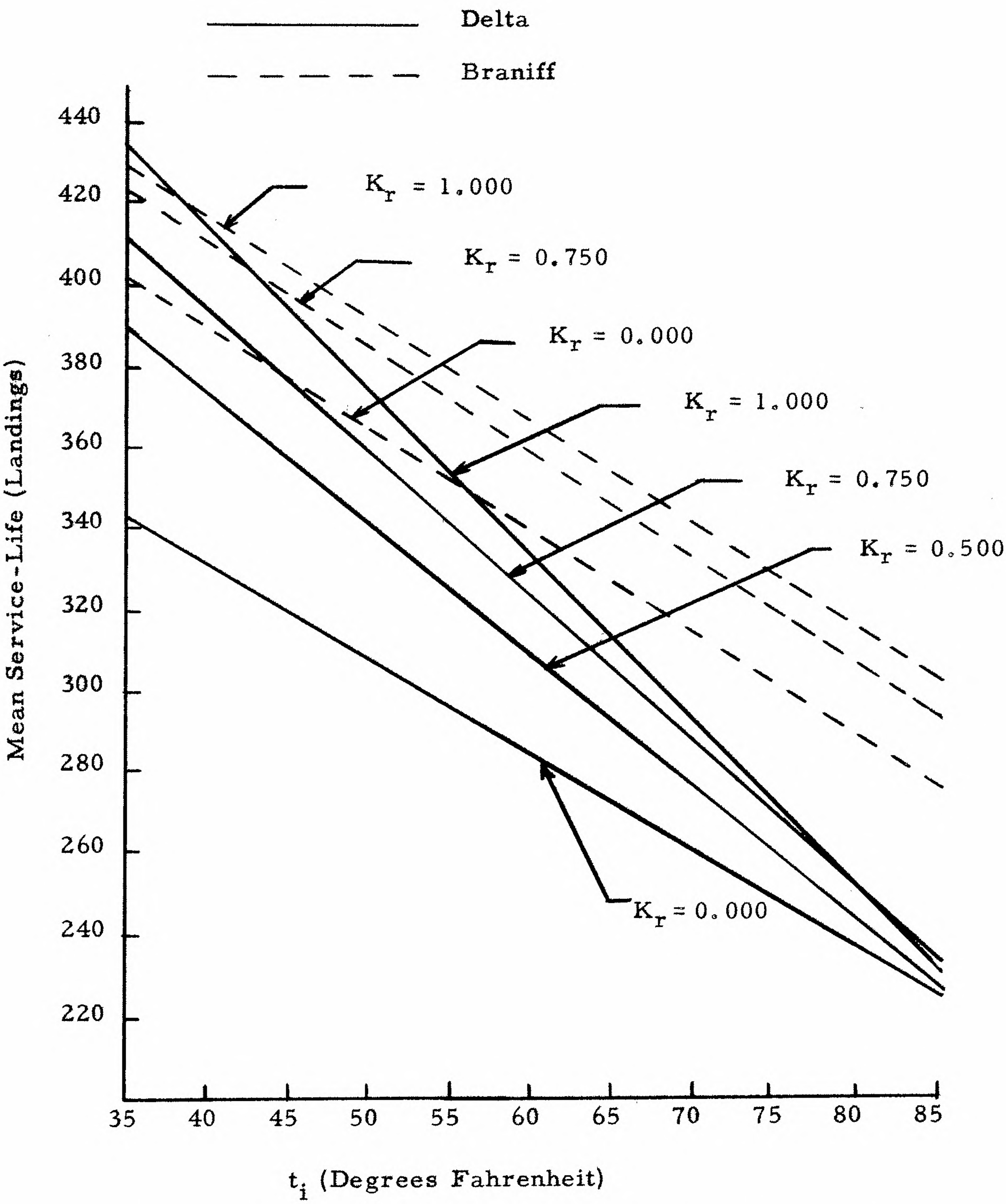


Figure 7. Graph of Three Variables; Proportion of Retreaded-Tire Utilization (K_r), Monthly Adjusted Atlanta Mean Temperature (t_i), and Mean Service-Life ($\bar{g}_i$) for Braniff and Delta

Table 34. Monthly Delta Aircraft Landings (L_i), Assembly Removals (R_i), and Landings per Assembly Removal (L_i/R_i) in the Period April 1957 Through April 1960

	1957			1958			1959			1960		
Month	L_i	R_i	L_i/R_i	L_i	R_i	L_i/R_i	L_i	R_i	L_i/R_i	L_i	R_i	L_i/R_i
January				7430	83	89.52	7296	76	96.00	7222	88	82.07
February				6885	76	90.59	6547	71	92.21	7373	94	78.44
March				7525	81	92.90	7804	89	87.69	7581	90	84.23
April	7291	109	66.89	7505	97	77.37	7558	108	69.98	7979	116	68.78
May	7802	106	73.60	7242	95	76.23	7478	112	66.77			
June	7501	114	65.80	6969	100	69.69	7204	117	61.57			
July	7592	123	61.72	7073	101	70.03	7494	124	60.44			
August	7765	111	69.95	7026	103	68.21	7725	118	65.47			
September	7180	98	73.27	7129	95	75.04	7298	102	71.55			
October	7454	101	73.80	7356	91	80.84	7538	104	72.48			
November	6844	102	67.10	6811	77	88.45	7018	102	68.80			
December	7138	73	97.78	7176	78	92.00	7370	86	85.70			

hypothesis was established that $\bar{g}_i/4$ is a good estimate of L_i/R_i. This hypothesis was based on the following relationship. Assume that L_i was estimated to be 9000 landings/assembly/aircraft/month and $\bar{g}_i$ was estimated to be 300 landings/assembly removal. It was previously shown that the aircraft wheel position does not affect assembly removal frequency; therefore, it would be expected that the number of monthly removals (R_i) would be equal to

$$\frac{4 \text{ assemblies/aircraft} \times 9000 \text{ landings/assembly/aircraft/month}}{300 \text{ landings/assembly removal}}$$

= 120 assembly removals/month

or equivalently

$$L_i/R_i = \bar{g}_i/4$$

A chi-square contingency table test was used to compare the L_i/R_i and $\bar{g}_i/4$ in Table 35. This test used the statistic

$$\chi^2 = \sum_{i=1}^{37} \frac{(O_i - E_i)^2}{E_i}$$

$$O_i = \bar{g}_i/4$$

$$E_i = L_i/R_i$$

The critical chi-square value for 36 degrees of freedom is

$$\chi^2_{.05} = 51.0$$

Table 35. Delta Monthly Mean Service-Life Divided by Four ($\bar{g}_i/4$) and Landings per Assembly Removal (L_i/R_i) for Period April 1957 Through April 1960

Month	1957		1958		1959		1960	
	L_i/R_i	$\bar{g}_i/4$	L_i/R_i	$\bar{g}_i/4$	L_i/R_i	$\bar{g}_i/4$	L_i/R_i	$\bar{g}_i/4$
January			89.52	91.42	96.00	96.36	82.07	85.70
February			90.59	97.20	92.21	99.39	78.44	80.37
March			92.90	87.39	87.69	88.40	84.23	83.23
April	66.89	74.30	77.37	84.92	69.98	78.42	68.78	73.39
May	73.60	68.52	76.23	81.74	66.77	70.55		
June	65.80	69.02	69.69	73.73	61.57	66.38		
July	61.72	64.19	70.03	69.21	60.44	60.15		
August	69.95	67.68	68.21	65.26	65.47	63.28		
September	73.27	70.18	75.04	71.45	71.55	66.87		
October	73.80	71.09	80.84	76.33	72.48	72.04		
November	67.10	73.60	88.45	81.01	68.80	69.90		
December	97.78	76.14	92.00	86.53	85.70	75.70		

The actual chi-square value was found to be

$$\chi^2 = 15.2$$

Since the actual chi-square value is less than the critical chi-square value at the five per cent level of significance, there is no reason to reject the hypothesis that $\bar{g}_i/4$ is a good estimate of L_i/R_i.

A regression line using the method of least squares was fitted to the Delta $\bar{g}_i/4$ and L_i/R_i data in Table 35. The least squares equation was found to be

$$y = 8.91 + .884x$$

where y = estimate of L_i/R_i

$x = \bar{g}_i/4$

The correlation coefficient r and $s_{y/x}$ were found to be .836 and 5.90, respectively. The hypothesis that $\rho = 0$ was tested, and the value of t was found to be 8.99. The critical value of t is equal to

$$t_{.05} = 2.03$$

Since the actual t is greater than the critical value of $t_{.05}$, the hypothesis that $\rho = 0$ was rejected. Therefore, it was assumed a linear relationship existed between L_i/R_i and $\bar{g}_i/4$.

<u>Analysis of Monthly Differences Between Mean Service-Life of All Assembly Removals and Landings per Assembly Removal</u>.--An analysis

of the monthly differences between $\bar{g}_i/4$ and L_i/R_i was conducted to determine if the $\bar{g}_i/4$ estimate of L_i/R_i could be improved. Two hypotheses were established in this analysis. The first was based on a temperature relationship, and the second was based on a chance-cause relationship.

The first hypothesis established was that the quantity $(\bar{g}_i/4 - L_i/R_i)$ is related to the quantity $(q_i - q_{i-1})$, where q_i is the average Atlanta temperature in the ith month. This hypothesis was based on the observation that when $(q_i - q_{i-1})$ was positive, the $(\bar{g}_i/4 - L_i/R_i)$ tended to be positive. Conversely, when $(q_i - q_{i-1})$ was negative, the $(\bar{g}_i/4 - L_i/R_i)$ tended to be negative.

Table 36 presents the Delta monthly $(\bar{g}_i/4 - L_i/R_i)$ and $(q_i - q_{i-1})$. A regression line using the method of least squares was fitted to this data. The least squares equation was found to be

$$y = -.21 + .357x$$

where y = estimate of $(\bar{g}_i/4 - L_i/R_i)$

$x = q_i - q_{i-1}$

The correlation coefficient r and $s_{y/x}$ were found to be .498 and 5.22, respectively. The hypothesis that $\rho = 0$ was tested, and the value of t was found to be 3.40. The critical value of t at the five per cent level of significance is

$$t_{.05} = 2.03$$

Table 36. Delta Monthly Differences Between Mean Service-Life Divided by Four and Landings per Assembly Removal ($\bar{g}_i/4 - L_i/R_i$); and Monthly Temperature Differences ($q_i - q_{i-1}$)

	1957		1958	
ith Month	$\bar{g}_i/4 - L_i/R_i$	$q_i - q_{i-1}$	$\bar{g}_i/4 - L_i/R_i$	$q_i - q_{i-1}$
January			1.90	-7.8
February			6.61	-1.1
March			-5.51	12.0
April	7.41	12.8	7.55	12.6
May	-5.08	6.5	5.51	8.3
June	3.22	7.0	4.04	6.3
July	2.47	1.2	-.82	1.8
August	-2.27	.5	-2.95	.1
September	-3.09	-6.2	-3.59	-4.9
October	-2.71	-14.9	-4.51	-8.0
November	6.50	-4.7	-7.74	-5.5
December	-21.64	-7.0	-5.47	-13.8

	1959		1960	
ith Month	$\bar{g}_i/4 - L_i/R_i$	$q_i - q_{i-1}$	$\bar{g}_i/4 - L_i/R_i$	$q_i - q_{i-1}$
January	.36	-.5	3.63	-2.1
February	7.18	5.5	1.93	-.4
March	.71	3.5	-1.00	-2.0
April	8.44	11.8	4.61	21.9
May	3.78	9.2		
June	4.81	4.4		
July	-.29	2.5		
August	-2.19	1.5		
September	-4.68	-7.3		
October	-.44	-8.5		
November	1.10	-13.0		
December	-10.00	-5.5		

Since the actual t was greater than the critical value of $t_{.05}$, the hypothesis that $\rho = 0$ was rejected at the five per cent level of significance. Therefore, it was assumed that a linear relationship existed between $(\bar{g}_i/4 - L_i/R_i)$ and $(q_i - q_{i-1})$. The negative of the regression equation estimate y will be added to $\bar{g}_i/4$ to improve the estimate of L_i/R_i.

The second hypothesis established was that the quantity $(\bar{g}_i/4 - L_i/R_i)$ is related to the quantity $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$. Table 37 presents the Delta $(\bar{g}_i/4 - L_i/R_i)$ and $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ data, with the April value of the latter assumed to be equal to 3.07. It was assumed that over a long period of time the sum of all monthly values of the quantity $(\bar{g}_i/4 - L_i/R_i)$ would tend towards zero.

The April 1957 value of $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ was not available from the data. The April 1957 value was assumed to be 3.07, since the addition of this constant provides for the sum of all monthly values of the quantity $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ to equal zero. Figure 8 is a graphical presentation of $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ for the period May 1957 through April 1960, and for the period April 1957 through April 1960 with the April 1957 value equal to 3.07.

The hypothesis that the quantity $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ is related to $(\bar{g}_i/4 - L_i/R_i)$ was based on the following reasoning. A chance-cause increase in assembly removals in the i-1 month would result in a decrease in assembly removals in the ith month. Conversely, a chance-cause decrease in assembly removals in the i-1 month would

Table 37. Delta Monthly Differences Between Mean Service-Life Divided by Four and Landings per Assembly Removal ($\bar{g}_i/4 - L_i/R_i$); and Monthly Delta Cumulative $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$

	1957		1958	
ith Month	$\bar{g}_i/4 - L_i/R_i$	$\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$	$\bar{g}_i/4 - L_i/R_i$	$\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$
January			1.90	-12.12
February			6.61	-10.22
March			-5.51	-3.61
April	7.41	3.07	7.55	-9.12
May	-5.08	10.48	5.51	-1.57
June	3.22	5.40	4.04	3.94
July	2.47	8.62	-.82	7.98
August	-2.27	11.09	-2.95	7.16
September	-3.09	8.82	-3.59	4.21
October	-2.71	5.73	-4.51	.62
November	6.50	3.02	-7.74	-3.89
December	-21.64	9.52	-5.47	-11.33

(Continued)

Table 37. (Continued) Delta Monthly Differences Between Mean Service-Life Divided by Four and Landings per Assembly Removal ($\bar{g}_i/4 - L_i/R_i$); and Monthly Delta Cumulative $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$

	1959		1960	
ith Month	$\bar{g}_i/4 - L_i/R_i$	$\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$	$\bar{g}_i/4 - L_i/R_i$	$\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$
January	.36	-16.00	3.63	-8.02
February	7.18	-16.44	1.93	-4.39
March	.71	-9.26	-1.00	-2.46
April	8.44	-8.55	4.61	-3.46
May	3.78	-.11		
June	4.81	3.67		
July	-.29	8.48		
August	-2.19	8.19		
September	-4.68	6.00		
October	-.44	1.32		
November	1.10	.88		
December	-10.00	1.98		

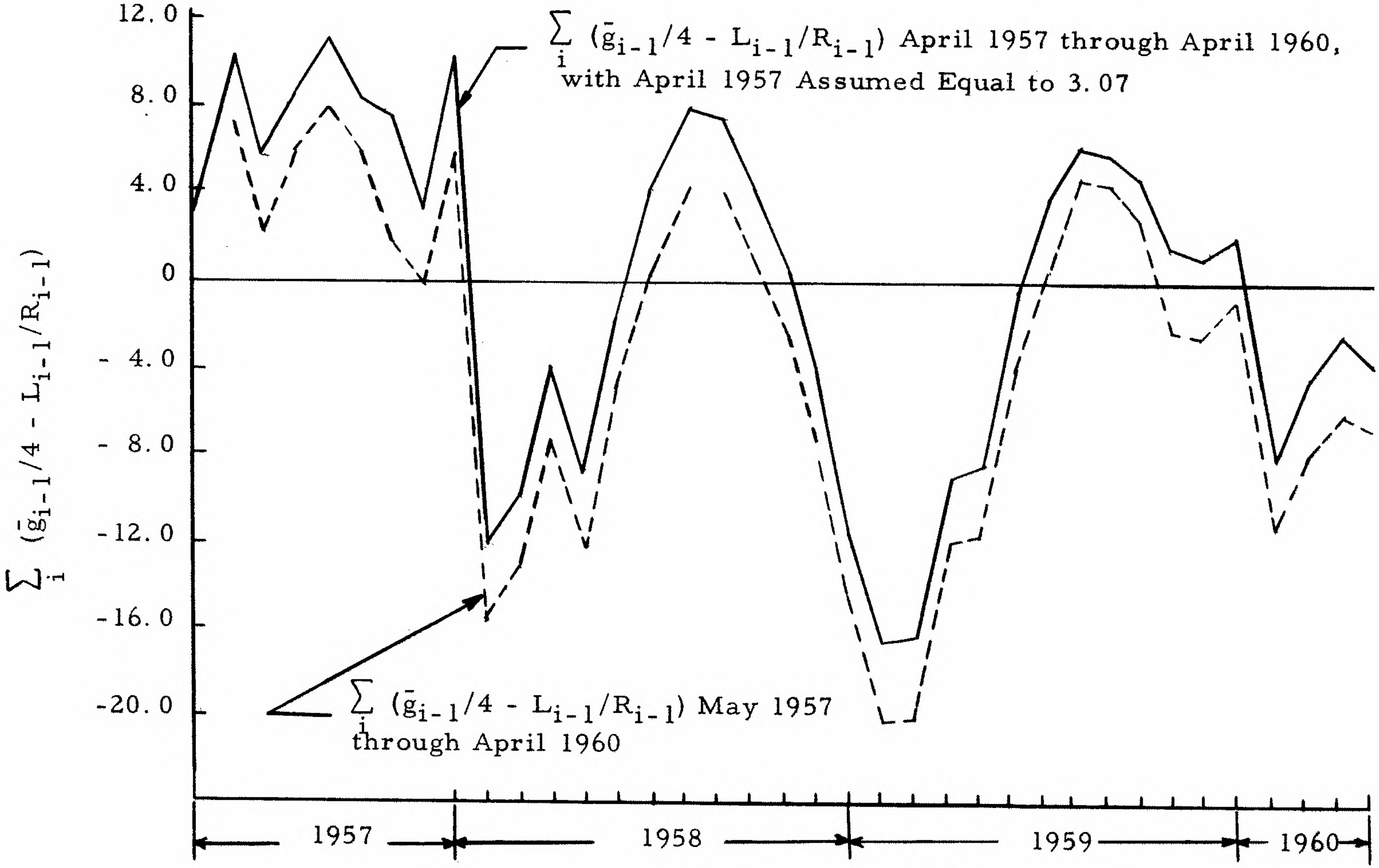


Figure 8. Variation of Monthly Delta Quantities $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ and $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1}) + 3.07$

result in an increase in assembly removals in the ith month. The former means that in the i-1 month, $\bar{g}_{i-1}/4$ would tend to overestimate L_{i-1}/R_{i-1}, and in the ith month $\bar{g}_i/4$ would tend to underestimate L_i/R_i. The hypothesis assumes that these increases and decreases in monthly assembly removals are random.

A regression line using the method of least squares was fitted to the data in Table 37. The least squares equation was found to be

$$y = -.26 - .301x$$

where y = estimate of $(\bar{g}_i/4 - L_i/R_i)$

$$x = \sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$$

The correlation coefficient r and $s_{y/x}$ were found to be -.406 and 5.45, respectively. The hypothesis that $\rho = 0$ was tested, and the value of t was found to be 2.59. The critical value of t at the five per cent level of significance is

$$t_{.05} = 2.03$$

Since the actual t was greater than the critical value of $t_{.05}$, the hypothesis that $\rho = 0$ was rejected at the five per cent level of significance. Therefore, it was assumed that a linear relationship existed between $(\bar{g}_i/4 - L_i/R_i)$ and $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$. The negative of the regression equation y will be added to $\bar{g}_i/4$ to improve the estimate of L_i/R_i.

To further substantiate the negative correlation between $(\bar{g}_i/4 - L_i/R_i)$ and $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$, the hypothesis was established that the sign of the quantity $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ is not related to the sign of the quantity $(\bar{g}_i/4 - L_i/R_i)$. This hypothesis was tested by a chi square test for independence (25). Table 38 presents the positive and negative sign relationships from the data in Table 37. The actual chi-square value was found to be

$$\chi^2 = 6.31$$

The critical chi-square value for one degree of freedom is

$$\chi^2_{.05} = 3.84$$

Since the actual chi-square value was greater than the critical $\chi^2_{.05}$ value, the hypothesis of independence was rejected at the five per cent level of significance. The rejection of this hypothesis further substantiated the existance of a relationship between $(\bar{g}_i/4 - L_i/R_i)$ and $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$.

An evaluation of the improvement in the estimate of L_i/R_i due to the application of these two correction factors was conducted. The evaluation criterion was taken to be the quantity

$$\sum_i (O_i - E_i)^2$$

where $O_i = \bar{g}_i/4$ or $\bar{g}_i/4$ + correction factor(s)

Table 38. Number of Monthly Positive and Negative Sign Relationships between Delta ($\bar{g}_i/4 - L_i/R_i$) and $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ for Period April 1957 Through April 1960

		$\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$		
		Positive	Negative	Total
$\bar{g}_i/4 - L_i/R_i$	Positive	7	12	19
	Negative	14	4	18
	Total	21	16	37

$$E_i = L_i/R_i$$

These correction factors were applied by the utilization of the $(q_i - q_{i-1})$ data in Table 36 and the $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ data in Table 37. The quantity $\sum_i (O_i - E_i)^2$ was computed for four cases. These cases were as follows:

1. $O_i = \bar{g}_i/4$
2. $O_i = \bar{g}_i/4 + .21 - .357 (q_i - q_{i-1})$
3. $O_i = \bar{g}_i/4 + .26 + .301 \sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$
4. $O_i = \bar{g}_i/4 + \left[.21 - .357 (q_i - q_{i-1})\right] + \left[.26 + .301 \sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})\right]$

The E_i was equal to L_i/R_i for all four cases. The results of these computations are shown in Table 39. These results show that the correction factors based upon $(q_i - q_{i-1})$ and $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ each decrease the quantity $\sum_i (O_i - E_i)^2$. However, the greatest

Table 39. Evaluation of the Improvement in the $\bar{g}_i/4$ Estimate of L_i/R_i by the Application of the Temperature and Chance-Cause Correction Factors

O_i	E_i	$\sum_i (O_i - E_i)$
$\bar{g}_i/4$	L_i/R_i	1,267.2
$\bar{g}_i/4 + .21 - .357\,(q_i - q_{i-1})$	L_i/R_i	954.8
$\bar{g}_i/4 + .26 + .301 \sum_i (\bar{q}_{i-1}/4 - L_{i-1}/R_{i-1})$	L_i/R_i	1,079.3
$\bar{g}_i/4 + .21 - .357\,(q_i - q_{i-1}) +$ $.26 + .301 \sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$	L_i/R_i	823.1

decrease was made when both of the factors were applied.

The O_i obtained from the application of both of the correction factors was designated $\bar{c}_i/4$. Table 40 presents the Delta monthly $\bar{c}_i/4$. A chi-square contingency table test was used with the $\bar{c}_i/4$ data in Table 40 and the L_i/R_i data in Table 35 to test the hypothesis that $\bar{c}_i/4$ is a good estimate of L_i/R_i.

Table 40. Adjusted Delta Monthly Mean Service-Life Divided by Four ($\bar{c}_i/4$)

ith Month	1957	1958	1959	1960
January		91.0	92.0	84.6
February		95.0	92.9	79.8
March		82.5	84.9	83.7
April	71.1	78.1	72.1	65.0
May	69.9	78.7	67.8	
June	68.6	73.2	66.4	
July	66.9	71.5	62.4	
August	71.3	68.0	65.8	
September	75.6	75.1	71.8	
October	78.6	79.9	75.9	
November	76.7	82.2	75.3	
December	82.0	88.5	78.8	

In this test

$$O_i = \bar{c}_i/4$$

$$E_i = L_i/R_i$$

The actual chi-square value was found to be

$$\chi^2 = 9.95$$

The critical chi-square value is

$$\chi^2_{.05} = 51.0$$

The actual chi-square is less than the critical $\chi^2_{.05}$ value. Hence, there was no reason to reject the hypothesis that $\bar{c}_i/4$ is a good estimate of L_i/R_i.

A regression line using the method of least squares was fitted to the $\bar{c}_i/4$ data in Table 40 and the L_i/R_i data in Table 35. The least squares equation was found to be

$$y = -10.64 + 1.135x$$

where y = estimate of L_i/R_i

$x = \bar{c}_i/4$

The correlation coefficient r and $s_{y/x}$ were found to be .900 and 4.70, respectively. The hypothesis that $\rho = 0$ was tested, and the value of t was found to be 12.21. The critical value of t at the five per cent level of significance is

$$t_{.05} = 2.03$$

Since the actual t is greater than the critical value of $t_{.05}$, the hypothesis that $\rho = 0$ was rejected. Hence, it was assumed that a linear relationship existed between $\bar{c}_i/4$ and L_i/R_i. A comparison of the $\bar{g}_i/4$ and

$\bar{c}_i/4$ relationships to L_i/R_i is shown in Table 41. The comparison in Table 41 shows that the quantity $\bar{c}_i/4$ is better than $\bar{g}_i/4$ for estimating L_i/R_i.

Forecasting of Landings per Removal.--It was shown that monthly mean service-life for new tires ($\bar{n}_i$), retreaded tires ($\bar{r}_i$), and all tires ($\bar{g}_i$) are related to t_i temperatures. It was also shown that the mean service-life of all tires divided by four ($\bar{g}_i/4$) can be corrected to provide a good estimate of the inventory demand parameter L_i/R_i. An example will now be shown to illustrate the use of this forecasting procedure.

On August 31, 1959, it is desired that there be a forecast of the Delta September 1959 value of L_i/R_i. The first step is to estimate $\bar{g}_i/4$. It is assumed that the proportion of tires used that are retreads (K_r) in September will be the same as in August. From Table 20 the K_r for August is found to be equal to 0.91. The equation derived previously to estimate Delta $\bar{g}_i$ is given by

$$\bar{g}_i = 424.2 - 2.32\ t_i + 155.9\ K_r - 1.85\ t_i\ K_r$$

The value of t_i (temperature) is given by

$$t_i = \frac{q_i + q_{i-1}}{2}$$

The value of q_{i-1} (August 1959) would be known on August 31 and is shown in Table 29 to be equal to 80.6 degrees. The value of q_i (September 1959) is not known and must be estimated. The best estimate of q_i is the Atlanta

Table 41. Comparison of Monthly $\bar{g}_i/4$ and $\bar{c}_i/4$ as Estimates of Landings per Removal (L_i/R_i)

Comparison Characteristic	$\bar{g}_i/4$	$\bar{c}_i/4$
Actual chi-square value from contingency table test	15.15	9.95
Regression equation	$L_i/R_i = 8.91 + .884\ (\bar{g}_i/4)$	$L_i/R_i = -10.64 + 1.135\ (\bar{c}_i/4)$
Correlation coefficient r	.836	.900
$s_{y/x}$	5.90	4.70

"record mean" temperature (m_i) for September. Table 1 shows the m_i for September to be equal to 73.3 degrees. The estimate of t_i is then given by

$$t_i = \frac{73.3 + 80.6}{2} = 76.95 \text{ degrees}$$

An estimate of $\bar{g}_i$ can be obtained from the $\bar{g}_i$ equation by using the K_r value of 0.91 and the t_i value of 76.95. The estimated value of $\bar{g}_i$ was found to be

$$\bar{g}_i = 258.1 \text{ landings per tire}$$

The estimated value of $\bar{g}_i/4$ was found to be

$$\bar{g}_i/4 = 64.5 \text{ landings per removal}$$

The $(q_i - q_{i-1})$ correction factor equation is given by

$$\bar{g}_i/4 - L_i/R_i = -.21 + .357 \, (q_i - q_{i-1})$$

By the use of the q_{i-1} and m_i shown above, the correction factor quantity is

$$\bar{g}_i/4 - L_i/R_i = -2.8 \text{ or equivalently}$$

$$L_i/R_i = \bar{g}_i/4 + 2.8$$

The $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ correction factor equation is given by

$$\bar{g}_i/4 - L_i/R_i = -.26 - .301 \sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$$

The September 1959 value of $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ is known on August 31, and from Table 37 is found to be equal to 6.0. By the use of this value, the correction quantity is

$$\bar{g}_i/4 - L_i/R_i = -1.1 \text{ or equivalently}$$

$$L_i/R_i = \bar{g}_i/4 + 1.1$$

The improved estimate $\bar{c}_i/4$ is then given by

$$\bar{c}_i/4 = 64.5 + 2.8 + 1.1 = 68.4 \text{ landings per removal}$$

From Table 34 the actual L_i/R_i for September 1959 was found to be equal to 71.6 landings per removal.

Daily Distribution of System Assembly Removals.--The previous section presented a procedure to forecast monthly landings per removal (L_i/R_i). A forecast of the number of monthly aircraft landings (L_i) can be made from flight schedules and historical data on the percentage of scheduled flights actually flown. A forecast of the number of monthly assembly removals (R_i) is then

$$R_i = \frac{L_i}{L_i/R_i}$$

The Delta daily distribution of R_i was studied for the year 1959.

The hypothesis was established that the observed daily assembly demand distributions were from Poisson distributed populations with the same means as the observed data. Table 42 shows the observed daily demand and expected daily demand from Poisson distributed populations for monthly, seasonal and annual data. Figure 9 compares the daily observed and expected demand for Season I and III data. The expected daily demands were calculated from Molina's (26) Poisson's Exponential Binomial Limit tables.

A chi-square goodness of fit test was used to test the hypothesis that the observed distributions were from Poisson distributed populations. The actual chi-square values and critical chi-square values at the five per cent level of significance are shown in Table 43. The hypothesis was rejected for the monthly August data at the five per cent level of significance. The tests of the other eleven months, three seasons, and complete year gave no reason to reject the hypothesis. On the basis of these tests it was assumed that the daily assembly demand distributions could be approximated by Poisson distributed populations.

Daily Distribution of Assembly Removals at Maintenance Bases and Line Maintenance Stations.--It was shown that the daily distribution of system assembly removals could be approximated by the Poisson distribution. It was next desirable to determine the daily distribution of assembly removals at maintenance bases and line maintenance stations.

Figure 10 presents a flow chart of the movement of wheels, tires,

Table 42. Delta 1959 Daily Observed Assembly Demand and Daily Expected Assembly Demand from Poisson Distributed Populations

Number of Removals	Observed	Expected from Poisson Distribution	Number of Removals	Observed	Expected from Poisson Distribution
January			February		
0	2	2.7	0	3	2.3
1	7	6.6	1	4	5.7
2	8	8.0	2	5	7.2
3	6	6.6	3	9	6.0
4	5	4.0	4	5	3.7
5	3	3.2	5	2	3.0
March			April		
0	2	1.7	0	2	.8
1	8	4.9	1	1	2.9
2	6	7.2	2	6	5.3
3	3	6.9	3	7	6.4
4	7	5.0	4	5	5.7
5	2	2.9	5	5	4.1
6	0	1.4	6	1	2.5
7	2	.6	7	1	1.3
8	1	.3	8	2	.9

(Continued)

Table 42. (Continued) Delta 1959 Daily Observed Assembly Demand and Daily Expected Assembly Demand from Poisson Distributed Populations

Number of Removals	Observed	Expected from Poisson Distribution	Number of Removals	Observed	Expected from Poisson Distribution
May			June		
0	1	.8	0	0	.6
1	6	3.0	1	3	2.4
2	3	5.5	2	5	4.6
3	6	6.6	3	6	6.0
4	2	5.9	4	6	5.9
5	8	4.3	5	4	4.6
6	2	2.6	6	3	3.0
7	2	1.3	7	1	1.7
8	1	1.0	8	1	.8
			9	1	.6
July			August		
0	1	.6	0	0	.7
1	3	2.3	1	4	2.6
2	4	4.5	2	8	5.0
3	4	6.1	3	8	6.3
4	8	6.1	4	1	6.0
5	4	4.8	5	1	5.6
6	3	3.2	6	5	2.9
7	2	1.8	7	1	1.6
8	1	.9	8	1	.7
9	1	.7	9	0	.3
			10	2	.2

(Continued)

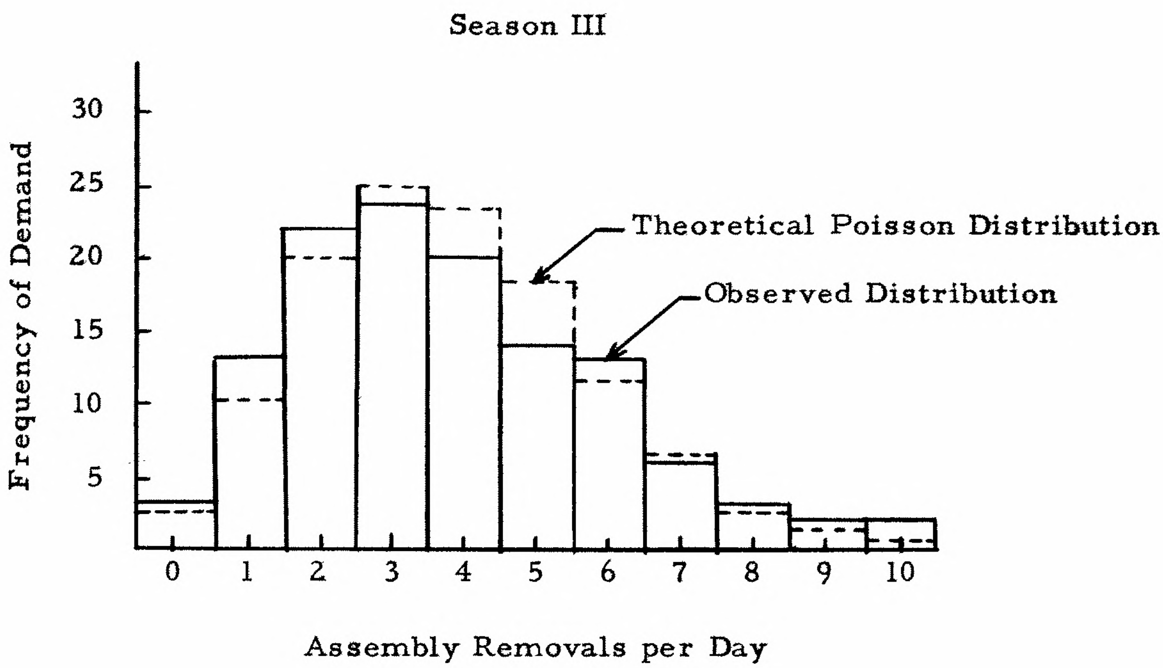
Table 42. (Continued) Delta 1959 Daily Observed Assembly Demand and Daily Expected Assembly Demand from Poisson Distributed Populations

Number of Removals	Observed	Expected from Poisson Distribution	Number of Removals	Observed	Expected from Poisson Distribution
September			October		
0	2	1.0	0	0	1.0
1	3	3.4	1	6	3.5
2	5	5.8	2	5	6.0
3	6	6.6	3	5	6.8
4	5	5.6	4	7	5.8
5	5	3.8	5	4	3.9
6	2	2.1	6	3	2.2
7	2	1.7	7	1	1.8
November			December		
0	2	1.0	0	2	1.9
1	4	3.4	1	5	5.3
2	4	5.8	2	9	7.4
3	8	6.6	3	6	6.9
4	4	5.6	4	3	4.8
5	3	3.8	5	4	2.7
6	3	2.1	6	1	1.3
7	1	1.0	7	1	.8
8	0	.4			
9	0	.2			
10	1	.1			

(Continued)

Table 42. (Continued) Delta 1959 Daily Observed Assembly Demand and Daily Expected Assembly Demand from Poisson Distributed Populations

Number of Removals	Observed	Expected from Poisson Distribution	Number of Removals	Observed	Expected from Poisson Distribution
Season I			Season II		
0	9	8.1	0	5	3.7
1	24	22.0	1	17	12.9
2	28	29.6	2	18	22.6
3	24	26.7	3	26	26.3
4	20	18.0	4	18	23.0
5	11	9.7	5	20	16.1
6	1	4.4	6	9	9.4
7	3	1.7	7	5	4.7
8	1	.8	8	3	2.1
			9	0	.8
			10	1	.4
Season III			Annual		
0	3	2.7	0	17	13.5
1	13	10.4	1	54	44.4
2	22	19.7	2	68	73.3
3	24	25.0	3	74	80.6
4	20	23.7	4	58	66.5
5	14	18.0	5	45	43.9
6	13	11.4	6	23	24.2
7	6	6.2	7	14	11.4
8	3	4.9	8	7	4.7
9	2	1.2	9	2	1.7
10	2	.7	10	3	.8



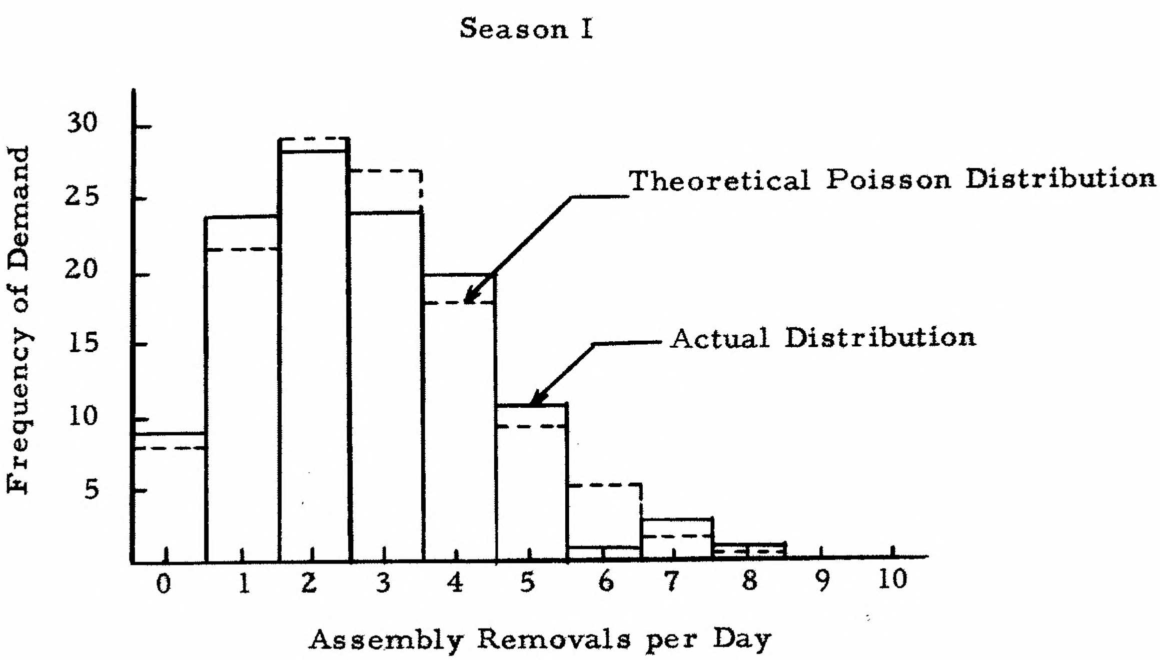


Figure 9. Actual and Theoretical Frequency Distributions of Delta Daily Assembly Demand

Table 43. Results of Chi-square Tests of Hypothesis that the Delta System Daily Assembly Demand Distribution is from a Poisson Distributed Population

Period Tested	Actual Chi-square Value	$\chi^2_{.05}$
January	.63	9.49
February	3.71	9.49
March	5.38	9.49
April	.68	9.49
May	8.04	9.49
June	.16	11.07
July	1.88	11.07
August	11.36	11.07
September	.68	9.49
October	1.40	9.49
November	2.38	9.49
December	1.59	9.49
Season I	1.54	11.07
Season II	5.10	14.07
Season III	3.57	12.59
Annual	8.85	14.07

and assemblies through the Delta system. The two Delta maintenance bases for the Convair 340/440 are Atlanta, Georgia, and Dallas, Texas. Each aircraft must return to either Atlanta or Dallas every 130 flight hours for an eight to ten hour secondary maintenance check. Each aircraft also must return to Atlanta every 2000 flight hours for a major maintenance check. The Atlanta maintenance base receives all unserviceable assemblies from the line maintenance stations and all unserviceable tires from the Dallas maintenance base. The unserviceable assemblies are disassembled, and the unserviceable tires to be

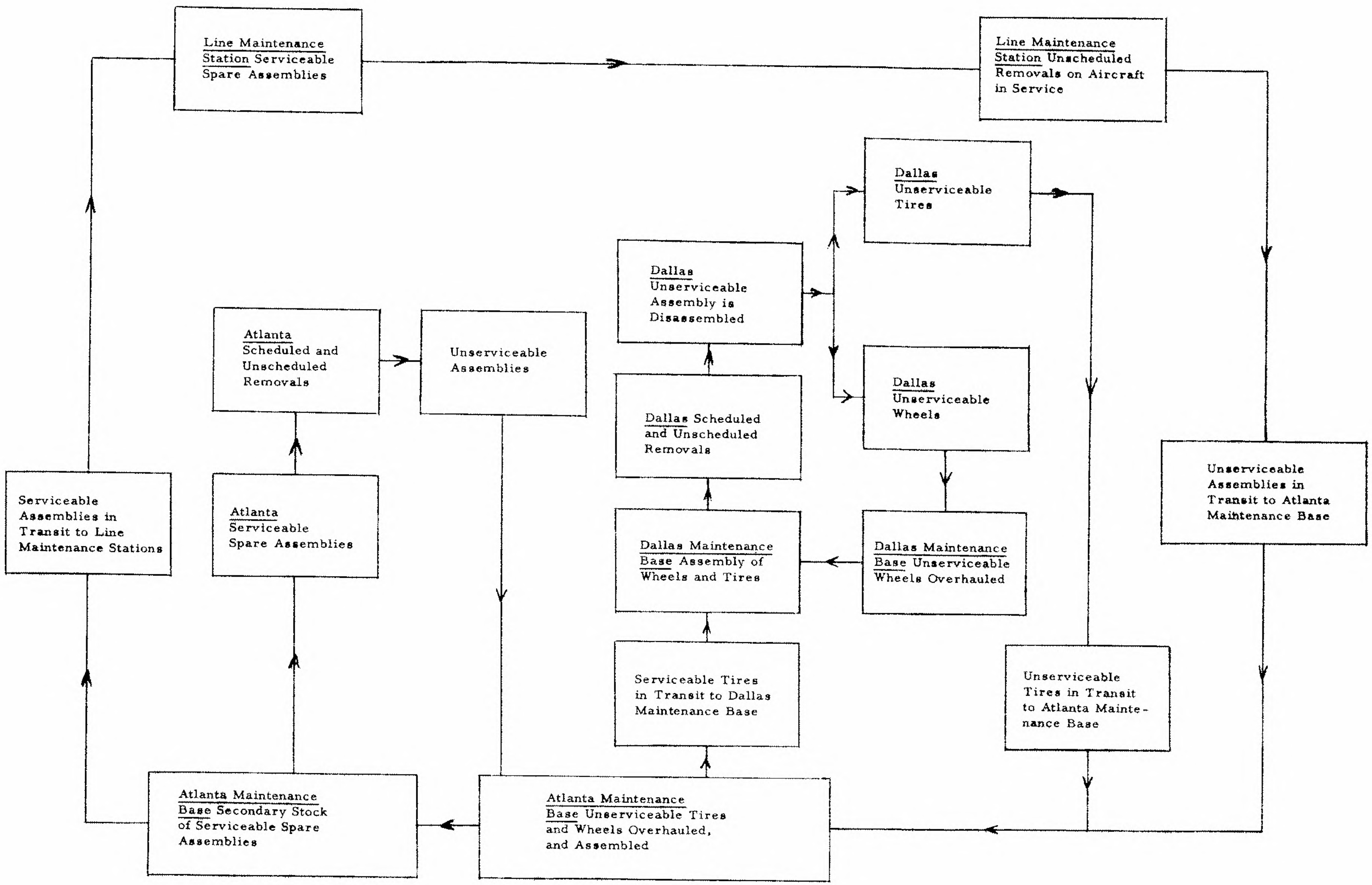


Figure 10. Flow Chart of Delta Wheels, Tires, and Assemblies

retreaded are sent to a retreading agency in Atlanta. The unserviceable tires from the Dallas maintenance base are also sent to the retreading agency in Atlanta via the Atlanta maintenance base. Newly retreaded tires are returned to the Atlanta maintenance base where they are mounted on wheels or forwarded to the Dallas maintenance base. The tires forwarded to the Dallas maintenance base are mounted on overhauled wheels and the assembly is placed into stock. At the Atlanta maintenance base the tires are mounted on overhauled wheels, and the assembly is placed into an Atlanta secondary inventory which is used to replenish line-maintenance-station and Atlanta maintenance base assembly inventories.

It is a Delta policy to remove assemblies prematurely at the maintenance bases and larger line maintenance stations, whenever possible. These premature removals primarily are made when an aircraft completes a flight at a large line maintenance station, or when the aircraft returns to either the Atlanta or Dallas maintenance base for a maintenance check. It is Delta's objective in making these premature removals to minimize the number of removals necessary at the smaller line maintenance stations, because these smaller stations do not have as great a maintenance capability as the maintenance bases and larger line maintenance stations. The stations classed as smaller line maintenance stations are generally the intermediate stations on flights where a removal is likely to result in a short flight delay if the spare assembly is available, or in a long flight delay if the spare assembly must be procured.

Table 44 shows the station codes of the stations served in 1959 by the Delta Convair 340/440. Table 45 shows the distribution of Delta 1959 monthly assembly removals by the station performing the removal. The stations were divided into five classes for the purpose of evaluating the station daily assembly demand distribution. These classes were:

1. Atlanta maintenance base.
2. Dallas maintenance base.
3. Chicago Midway line maintenance station.
4. Line maintenance stations with all landings resulting in flight terminations, other than Chicago Midway.
5. Line maintenance stations with landings resulting in both non-terminating flight stops and flight terminations, and line maintenance stations with all landings resulting in non-terminating flight stops.

Table 46 shows the stations included in classes four and five above. The above five station classes were chosen to represent various station maintenance capability characteristics. Atlanta and Dallas are the Delta maintenance bases, and together performed 50.8 per cent of the removals in 1959. Chicago Midway is a large line maintenance station, and performed the most removals of any Delta line maintenance station in 1959. The stations in class four all stock spare assemblies, and all

Table 44. Station Codes Used to Designate the Stations Served in 1959 by the Delta Convair 340/440

Station	Code
1. Augusta, Georgia	AGS
2. Atlanta, Georgia	ATL
3. Birmingham, Alabama	BHM
4. Beaumont/Port Arthur, Texas	BPT
5. Baton Rouge, Louisiana	BTR
6. Columbia, South Carolina	CAE
7. Chattanooga, Tennessee	CHA
8. Charleston, South Carolina	CHS
9. Charlotte, North Carolina	CLT
10. Columbus, Georgia	CSG
11. Cincinnati, Ohio	CVG
12. Dallas, Texas	DAL
13. Dayton, Ohio	DAY
14. Washington, D. C.	DCA
15. Detroit, Michigan	DTW
16. Evansville, Indiana	EVV
17. Newark, New Jersey	EWR
18. Fort Wayne, Indiana	FWA
19. Hot Springs, Arkansas	HOT
20. Houston, Texas	HOU
21. Indianapolis, Indiana	IND
22. Jackson, Mississippi	JAN
23. Lexington, Kentucky	LEX
24. Little Rock, Arkansas	LIT
25. Macon, Georgia	MCN
26. Midway Airport, Chicago, Illinois	MDW
27. Meridian, Mississippi	MEI
28. Memphis, Tennessee	MEM
29. Montgomery, Alabama	MGM
30. Kansas City, Missouri	MKC
31. Monroe, Louisiana	MLU
32. New Orleans, Louisiana	MSY
33. O'Hare Airport, Chicago, Illinois	ORD
34. Philadelphia, Pennsylvania	PHL
35. Paducah, Kentucky	PUK
36. Savannah, Georgia	SAV

(Continued)

Table 44. (Continued) Station Codes Used to Designate the Stations Served in 1959 by the Delta Convair 340/440

Station	Code
37. Louisville, Kentucky	SDF
38. Springfield, Missouri	SGF
39. Shreveport, Louisiana	SHV
40. Brunswick, Georgia	SSI
41. Saint Louis, Missouri	STL
42. Toledo, Ohio	TOL
43. Knoxville, Tennessee	TYS
44. Ciudad Trujillo, Dominican Republic	CTJ
45. Havana, Cuba	HAV
46. Montego Bay, Jamaica	MBJ
47. Port Au Prince, Haiti	PAP
48. San Juan, Puerto Rico	SJU

landings at these stations result in flight terminations. The stations in class five include stations that stock spare assemblies and stations that do not. All of the stations in class five have landings that are non-terminating flight stops, although several class five stations have up to 35 per cent of their landings resulting in flight terminations.

The hypothesis was established that the observed daily assembly demand distributions for each of the five classes of stations were from Poisson distributed populations with the same means as the observed data. A chi-square goodness of fit test was used to test this hypothesis using monthly, seasonal, and annual data. The actual chi-square values and critical chi-square values at the five per cent level of significance

Table 45. Number of Monthly Assembly Removals Performed by Delta Stations in 1959

Station	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
AGS					1								1
ATL	37	31	31	49	37	38	52	33	24	28	34	27	421
BHM					1								1
BPT					1								1
CAE	1			1	1	1	2	4	4	2	2		18
CHS		1		1	1		1	5	3	2	1	1	16
CVG				2		1		1		2		1	7
DAL	18	14	15	13	22	17	21	15	15	12	17	14	193
DTW	1	1	1	3	5	4	9	6	6	11	4	3	54
EVV		1	1	1	1	1	4	2	2	5	6	2	26
EWR		1				2	2		1				6
HOU	2	1	2	2	1	3	1	1	1	6	2	3	25
IND	1	3	1	7	4	10	6	7	2		4	2	47
JAN					1				1	1		1	4
LEX											1		1
LIT			1				1						2
MDW	4	8	19	16	13	23	16	28	18	18	12	14	189
MEI	1				1								2
MEM	5	5	4	3	10	8	3	4	4	4	1	5	56
MKC			2		3	1			4		2		12
MSY	2	3	4	3	3	2		10	12	7	11	8	65
ORD	2		2	4	4	3	3		3	4	1	5	31
SAV							1						1
SDF						1							1
SHV		1	2	3	1	2			1		2		12
SJU		1			1			1		1	1		5
STL	2		4				2				1		9
TYS										1			1
Unknown								1	1				2
Total	76	71	89	108	112	117	124	118	102	104	102	86	1,209

Table 46. Delta Line Maintenance Stations Included in Station Classes Four and Five

Class Four. Line Maintenance Stations with All Landings Resulting in Flight Terminations

CHS	HOU	ORD
DTW	MKC	SAV
EWR	MSY	SJU

Class Five. Line Maintenance Stations with Landings Resulting in Both Non-terminating Flight Stops and Flight Terminations, and Line Maintenance Stations with All Landings Resulting in Non-terminating Flight Stops

AGS	CSG	IND
BHM	CVG	JAN
BPT	DAY	LEX
BTR	DCA	LIT
CAE	EVV	MCN
CHA	FWA	MEI
CLT	HOT	MEM
MGM	SGF	TYS
MLU	SHV	CTJ
PHL	SSI	HAV
PUK	STL	MBJ
SDF	TOL	PAP

are shown in Table 47. These results show that the hypothesis was rejected in two of the 60 monthly tests, two of the 15 seasonal tests, and two of the five annual tests. The hypothesis was not rejected in the other 74 tests. Since the hypothesis was not rejected in 92.5 per cent of the tests, it was assumed that the observed daily assembly demands

Table 47. Results of Chi-square Tests of Hypothesis that the Delta Station Class Daily Assembly Demand Distributions are from Poisson Distributed Populations

1959 Time Period	Station Class 1 Actual χ^2	Station Class 1 $\chi^2_{.05}$	Station Class 2 Actual χ^2	Station Class 2 $\chi^2_{.05}$	Station Class 3 Actual χ^2	Station Class 3 $\chi^2_{.05}$
January	3.67	5.99	1.66	3.84	.30	3.84
February	1.11	5.99	2.60	3.84	0.00	3.84
March	4.72	5.99	1.93	3.84	.22	3.84
April	5.87	7.81	2.05	3.84	.20	3.84
May	3.49	5.99	1.26	3.84	.02	3.84
June	4.54	5.99	.53	3.84	4.40	5.99
July	1.17	7.81	.09	3.84	.73	3.84
August	.73	5.99	.27	3.84	10.13*	5.99
September	.25	3.84	.28	3.84	3.66	3.84
October	.75	3.84	1.34	3.84	.54	3.84
November	.71	5.99	2.41	3.84	.02	3.84
December	.94	3.84	5.22*	3.84	2.06	3.84
Season I	6.84*	5.99	10.69*	3.84	.86	3.84
Season II	7.34	7.81	2.48	3.84	.12	3.84
Season III	2.57	7.81	1.63	5.99	5.52	5.99
Annual	10.23*	7.81	4.57	5.99	6.89*	5.99

1959 Time Period	Station Class 4 Actual χ^2	Station Class 4 $\chi^2_{.05}$	Station Class 5 Actual χ^2	Station Class 5 $\chi^2_{.05}$
January	.21	3.84	.23	3.84
February	1.63	3.84	.43	3.84
March	2.18	3.84	3.68	3.84
April	1.70	3.84	1.87	3.84
May	1.64	3.84	.12	5.99
June	.04	3.84	.76	5.99
July	2.35	3.84	.54	5.99
August	1.48	3.84	.14	3.84
September	.46	5.99	.66	3.84
October	5.18	5.99	.03	3.84
November	.30	3.84	.26	3.84
December	.88	3.84	.92	3.84
Season I	1.03	3.84	1.90	3.84
Season II	2.30	5.99	.71	3.84
Season III	.17	5.99	1.26	3.84
Annual	3.75	5.99	2.57	5.99

* Results where actual χ^2 is greater than $\chi^2_{.05}$

for each of the five classes of stations could be approximated by Poisson distributed populations.

It was shown that the assembly demand for the system is a function of the number of aircraft landings (L) and landings per removal (L/R). It also was shown that the system L/R is inversely related to seasonal temperature fluctuations. From this it was assumed that a station's L/R is also inversely related to seasonal temperature fluctuations. Table 48 shows the L/R in Seasons I, II, and III for the five station classes. This table shows that for each station class the L/R decreases from Season I to II, and II to III. The only exception was in class one, where the L/R in Season III was greater than the L/R in Season II. These observations are in agreement with the assumption that station L/R is inversely related to seasonal temperature fluctuations, since seasonal temperatures increase from Season I to II, and II to III. Table 46 also shows that within seasons the L/R between station classes differs greatly (e. g. in Season I station-class-two L/R was 5. 8 and station-class-five L/R was 494. 6). A study was conducted and a combination of three factors was found to be related to these differences in L/R among station classes. These factors are as follows:

1. Maintenance capability of the station.
2. Proportion of station landings resulting in flight terminations.
3. Management policies.

Table 48. Delta 1959 Landings per Removal (L/R) by Station Classes and Seasons

Station Class	Season I L	Season I R	Season I L/R	Season II L	Season II R	Season II L/R
1	2,259	126	17.9	2,393	148	16.2
2	353	61	5.8	348	64	5.4
3	1,431	45	31.8	1,480	59	25.1
4	3,214	46	69.9	3,363	84	40.0
5	21,762	44	494.6	22,008	71	310.0
System	29,017	322	90.1	29,592	426	69.5

Station Class	Season III L	Season III R	Season III L/R	Annual L	Annual R	Annual L/R
1	2,406	147	16.4	7,056	421	16.8
2	351	68	5.2	1,052	193	5.5
3	1,573	85	18.5	4,484	189	23.7
4	3,321	85	39.1	9,898	215	46.0
5	22,070	76	290.4	65,840	191	344.7
System	29,721	461	64.5	88,330	1,209	73.1

Some other factors which might have contributed to these differences, but were not evaluated are as follows:

1. Station runway surface composition.
2. Station runway length.
3. Station prevailing winds.
4. Station parking apron layout.

The effect of the station maintenance capability is shown in

Table 48. Maintenance capability refers to the number of spares available, the size of the maintenance facility, and the number of available mechanics who are qualified to perform assembly removals. Station classes one (Atlanta) and two (Dallas) are maintenance bases with the greatest maintenance capability, and Table 48 shows they have the lowest L/R. Station classes three and four have a greater maintenance capability than station class five, and the L/R for classes three and four is correspondingly lower than the L/R for class five. Although several of the stations in class five (e. g., CAE, IND, and MEM) have a greater maintenance capability than some of the stations in class four, only 14 of the 36 class five stations stocked spare assemblies while all of the class four stations stocked assemblies. These data indicate that station L/R is influenced by the capability of the stations to perform removals.

Table 48 also shows the effect on L/R of the proportion of station landings resulting in flight terminations. Although Atlanta and Dallas are both maintenance bases, the L/R for Dallas was approximately one-third of the L/R for Atlanta. Part of this difference can be attributed to the fact that in 1959 all Dallas landings resulted in flight terminations, while only 67 per cent of Atlanta landings resulted in flight terminations. It would be expected that the non-terminating landings in Atlanta, with limited aircraft ground time, would permit only the essential removals and very few premature removals. This factor would tend to cause the L/R in Atlanta to be greater than the L/R in Dallas.

The 1959 L/R of the class five stations that stocked spares was compared to the 1959 L/R of station classes three and four. The L/R for these class five stations was 219.4. The L/R for the class three and four stations were 23.7 and 46.0, respectively. These class five stations had from zero to 35 per cent of their landings resulting in flight terminations, while all of the station class three and four landings resulted in flight terminations. These data also indicate that an increase in the proportion of station landings resulting in flight terminations tends to decrease station L/R.

Management policy also affects station L/R. For example, there are two Delta stations which are connected by several flights. One station, a class five station, stocked spares and had 35 per cent of its landings result in flight terminations. The other station, a class four station, had all its landings result in flight terminations. The former had a 1959 L/R of 157.3, and the latter a 1959 L/R of 85.8. The 1959 L/R for all class five stations with spares was 219.4 and the 1959 L/R for all class four stations was 46.0. This indicated the class five station was making more removals, and the class four station was making fewer removals, than stations in their same class. This relationship was explained by a Delta maintenance official. He stated that the class five station has a very experienced maintenance crew, which was very conscientious in prematurely removing assemblies that were nearing minimum serviceable limits. Therefore, several premature removals were performed at the class five station. He further stated that the class four station has a smaller maintenance crew, which also has the collateral assignment of assembling rubber hoses for other Delta stations. Therefore, very few premature removals were performed at the class four station. Several other examples of management policy affecting station L/R also were mentioned by this Delta official.

Individual station L/R is a function of these and other factors. However, monthly or seasonal estimates of a station's L/R can be made from historical data. This estimated L/R divided into station daily landings (L) will yield a mean daily assembly demand, which can be assumed to be the mean daily assembly demand of a Poisson distributed population. The daily assembly demand distribution will be considered further in a subsequent section which considers inventory model construction.

Analysis of Monthly Mean Service-Life of Prematurely Removed Assemblies.--It has been stated that Delta policy causes removals to be made prematurely at maintenance bases and the larger line maintenance stations. It was desirable to determine how much assembly service-life was lost because of these premature removals. This analysis was conducted by comparing the Delta service-life of removals caused by wear at the Atlanta and Dallas maintenance bases, with the service-life of removals caused by wear at all other stations. This approach was based on the assumption that most premature removals are classified as being due to wear, since a premature removal generally is made when the tire approaches the minimum serviceable limit. It was also assumed that all wear removals at Atlanta and Dallas were premature removals, and all wear removals at other stations were caused by tires worn to minimum serviceable limits.

Table 49 shows the $\bar{w}_1$ (mean service-life of Delta new-tire wear removals at Atlanta or Dallas), $\bar{w}_2$ (mean service-life of new-tire wear removals at stations other than Atlanta or Dallas), $\bar{w}_3$ (mean service-life of Delta retreaded-tire wear removals at Atlanta or Dallas), and $\bar{w}_4$ (mean service-life of Delta retreaded-tire wear removals at stations other than Atlanta or Dallas).

An analysis of variance was conducted with the monthly new-tire and retreaded-tire data in Table 49. The hypotheses tested are as follows:

1. There is no difference between $\bar{w}_1$ and $\bar{w}_2$.
2. There is no difference between $\bar{w}_3$ and $\bar{w}_4$.
3. Monthly variability does not affect $\bar{w}_1$ and $\bar{w}_2$.
4. Monthly variability does not affect $\bar{w}_3$ and $\bar{w}_4$.

A separate analysis of variance was conducted for the new-tire data and the retreaded-tire data. The analysis of variance results are shown in Tables 50 and 51. These results reject all the hypotheses

Table 49. Delta 1959 Monthly Mean Service-Life (in Landings) of Wear Removals at Atlanta and Dallas ($\bar{w}_1$ and $\bar{w}_3$), and at Stations other than Atlanta and Dallas ($\bar{w}_2$ and $\bar{w}_4$)

	New Tires		Retreaded Tires	
Month	$\bar{w}_1$	$\bar{w}_2$	$\bar{w}_3$	$\bar{w}_4$
January	317.5	324.4	457.5	489.1
February	364.5	352.8	446.3	493.5
March	281.4	297.2	407.2	425.3
April	283.4	293.9	354.5	364.4
May	290.5	282.6	317.2	329.2
June	257.6	278.6	291.6	315.2
July	232.6	234.4	276.2	286.8
August	253.2	258.4	262.3	289.5
September	264.4	285.4	284.2	287.8
October	321.0	296.1	301.7	294.5
November	308.8	303.7	299.9	308.5
December	318.0	329.9	313.5	331.6
Annual	291.1	294.8	334.3	351.3

stated above at the five per cent level of significance, except the hypothesis that there is no difference between $\bar{w}_1$ and $\bar{w}_2$.

The rejection of the hypotheses of monthly variability is in agreement with the previous analyses of the effect of temperature fluctuations on mean service-life. The rejection of the hypothesis that there is no difference between $\bar{w}_3$ and $\bar{w}_4$, indicates that the $\bar{w}_4$ is significantly greater than $\bar{w}_3$. The analysis of variance did not reject the hypothesis that there is no difference between $\bar{w}_1$ and $\bar{w}_2$. The annual $\bar{w}_2$ was 1.0 per cent greater than the annual $\bar{w}_1$, and the annual $\bar{w}_4$ was

Table 50. Analysis of Variance Table for Monthly 1959 Delta New-Tire Mean Service-Life ($\bar{w}_1$ and $\bar{w}_2$)

Source	Sum of Squares	Degrees of Freedom	Mean Square	Variance Ratio	$F_{.05}$
Monthly effect	24,824.5	11	2,256.8	23.2	2.82
Station effect	82.5	1	82.5	.9	4.84
Residual	1,070.9	11	97.4		
Total					

Table 51. Analysis of Variance Table for Monthly 1959 Delta Retreaded-Tire Mean Service-Life ($\bar{w}_3$ and $\bar{w}_4$)

Source	Sum of Squares	Degrees of Freedom	Mean Square	Variance Ratio	$F_{.05}$
Monthly effect	112,301.9	11	10,209.3	100.8	2.82
Station effect	1,722.1	1	1,722.1	17.0	4.84
Residual	1,113.7	11	101.2		
Total	115,137.7				

5.1 per cent greater than the annual $\bar{w}_3$.

These results led to the conclusion that over five per cent of assembly service-life is lost in a premature removal, even though Table 49 shows a 1.0 and 5.1 per cent difference between $\bar{w}_1$ and $\bar{w}_2$, and $\bar{w}_3$ and $\bar{w}_4$, respectively. This conclusion considered the assumptions made that all wear removals in Atlanta and Dallas were premature removals, and all wear removals at other stations were caused by tires worn to minimum serviceable limits. This was not the true situation, because several of the wear removals at Atlanta and Dallas were caused by tires worn to minimum serviceable limits, and several of the wear removals at other stations were premature removals. This would tend to cause $\bar{w}_1$ and $\bar{w}_3$ to overestimate the actual mean service-life of Atlanta and Dallas premature removals classed as wear removals. Similarly, this would tend to cause $\bar{w}_2$ and $\bar{w}_4$ to underestimate the actual mean service-life of other station removals which were worn to serviceable limits and classed as wear removals.

Therefore, it was assumed that the 1.0 and 5.1 per cent differences were underestimates of the true differences between the service-life of prematurely removed assemblies and assemblies utilized to the end of their serviceable life.

<u>Flight Delays Resulting from Unserviceable Assemblies</u>.--The Delta daily "Flight Delay Report" showed that in 1959 there were 38 flight delays which resulted from an unserviceable assembly. These delays

varied from four minutes to four hours and 14 minutes. Thirty-two of these delays were at stations that stocked spare assemblies, and the delay time was only the time involved in the assembly removal. (A Delta official estimated that it takes approximately 20 minutes to replace an unserviceable assembly with a serviceable assembly.) These 32 delays had a mean delay time of 18.3 minutes. The other six delays were at stations that did not stock spare assemblies, and the delays were very long because an assembly had to be procured from another station. These six delays had a mean delay time of two hours and 25 minutes.

The 38 removals were due to the following removal causes:

	Number of Removals
1. Flat or leaking tire	24
2. Tire tread separation	6
3. Unserviceable wheel	3
4. Tire blowout	2
5. Tire tread worn	2
6. Worn spot on tire tread	1

The six removals at stations not stocking spare assemblies were caused by four flat tires and two tire blowouts. The Delta policy of prematurely removing assemblies was considered to be responsible for only six removals at stations not stocking spare assemblies. The effectiveness of this policy is evident when it is considered that stations not stocking spare assemblies accounted for 28.4 per cent of 1959 landings, but only 0.5 per cent of 1959 assembly removals. There were no flight delays reported due to a shortage of spare assemblies at stations which stocked spare assemblies.

An investigation of the service-life obtained from the assemblies causing delays was conducted. The service-life of these new and retreaded-tire removals was compared to the $\bar{n}$ and $\bar{r}$, respectively, of the month in which the removal was performed. The monthly $\bar{n}$ and $\bar{r}$ are shown in Table 20. It was found that the one new tire removed had a service-life which was 104.2 landings less than the monthly $\bar{n}$, and the 37 retreaded tires removed had an average service-life of 108.8 landings less than the monthly $\bar{r}$. This indicated that these delays were caused by assemblies becoming unserviceable early in service-life.

It was noted that of the 35 delay removals caused by the tire becoming unserviceable, 34 were retreaded tires and only one was a new tire. During 1959 Delta used 230 new tires, 968 retreaded tires, and 11 tires of which it was not known whether they were new or retreaded. The hypothesis was established that the proportion of delays caused by retreaded tires is equal to the proportion of retreaded-tire utilization.

The proportion of delays caused by retreaded tires was

$$K_d = .971$$

The proportion of retreaded-tire utilization was

$$K_r = .808$$

The statistic used was (27)

$$Z = \frac{K_d - K_r}{s_k}$$

where

$$k = \frac{\frac{34}{35} + \frac{968}{1198}}{} = .813$$

and $$s_k = \sqrt{(.813)(.187)(\frac{1}{35} + \frac{1}{1198})} = .068$$

The value of Z was found to be

$$Z = 2.40$$

The critical value of Z at the five per cent level of significance is

$$Z_{.05} = 1.98$$

Since the actual Z is greater than the critical $Z_{.05}$ value, the hypothesis was rejected, and it was concluded that K_d was greater than K_r at the five per cent level of significance. This means that retreaded tires were causing a greater proportion of flight delays than would be expected from the proportion of retreaded-tire utilization.

From the classification of delay removal causes it is seen that wear removals caused only three per cent of all delays due to assembly removals. It would be desirable for commercial airlines to have all removals caused by wear. This would increase assembly service-life and facilitate premature removals. The latter would be desirable because it would minimize the number of removals at the smaller line maintenance stations. Therefore, it was desirable to compare the proportion of new tires removed for wear and the proportion of retreaded tires removed for wear.

The hypothesis was established that the proportion of new tires removed for wear is equal to the proportion of retreaded tires removed for wear. Table 52 shows the proportion of new and retreaded tires removed for wear (V_n and V_r) for Delta in 1959 and for Braniff in the period January 1957 through October 1959. The statistic used to test this hypothesis was

$$Z = \frac{V_n - V_r}{s_v}$$

where

Delta $\quad v = \dfrac{191 + 630}{230 + 968} = .685$

Braniff $\quad v = \dfrac{491 + 1098}{737 + 1950} = .591$

Delta $\quad s_v = \sqrt{(.685)(.315)(\frac{1}{230} + \frac{1}{968})} = .033$

Table 52. Number of New and Retreaded Tires Removed for Wear by Delta in 1959 and by Braniff in Period January 1957 Through October 1959

	Delta	Braniff
Number of new tires removed for wear	191	491
Total number of new tires removed	230	737
Proportion of new tires removed for wear (V_n)	.830	.666
Number of retreaded tires removed for wear	630	1,098
Total number of retreaded tires removed	968	1,950
Proportion of retreaded tires removed for wear (V_r)	.651	.563

Braniff $$s_v = \sqrt{(.591)(.409)(\frac{1}{737} + \frac{1}{1950})} = .019$$

The value of the Delta Z was found to be

$$Z = 5.42$$

The value of the Braniff Z also was found to be

$$Z = 5.42$$

Since both the Delta and Braniff Z are greater than the critical $Z_{.05}$ value, the hypothesis was rejected at the five per cent level of significance. It was concluded that the proportion of new tires removed for wear (V_n) was significantly greater than the proportion of retreaded tires removed for wear (V_r). This means that new tires are more likely to wear out, and therefore cause fewer delays because they can be more readily removed prematurely.

Development of Inventory Decision Models. --The preceding analyses provided information for the inventory decision models developed in this section. These decision models are based on Delta data and the characteristics of the Delta inventory system. These models consider three inventory problems, which are as follows:

1. Assembly-stocking policies for maintenance bases and line maintenance stations
2. Assembly-stocking policies for the Atlanta secondary inventory

3. Tire-stocking policies at the Atlanta maintenance base

The three inventory variables considered in the decision model for stocking policies at maintenance bases and line maintenance stations are as follows:

1. Inventory demand parameter
2. Cost of carrying inventory
3. Cost of inventory shortage

The inventory demand parameter (landings per removal) has been discussed in previous sections of this study. It was shown that the individual maintenance-base and line-maintenance-station L/R is variable and depends on a combination of several factors. Therefore, it is necessary to use historical L/R data to estimate individual maintenance-base and line-maintenance-station mean daily assembly demand ($\bar{G}$). A station's $\bar{G}$ is obtained by dividing daily station landings by the station L/R. A station's $\bar{G}$ tends to be greater in the summer months than in the winter months, since station L/R was found to be inversely related to climatological temperatures.

The cost of carrying assembly inventories at the stations was estimated to be 20 per cent per year of the original cost of the assembly. The cost of carrying inventory includes the costs associated with storage, interest on capital invested in inventory, obsolescence, handling and distribution, taxes, depreciation, and insurance. The exact costs associated with these factors were beyond the scope of this study. The 20 per cent estimate was based on the 10-25 per cent inventory carrying-cost estimates presented by Whitin (28). The difficulty in estimating inventory carrying costs is shown by examination of the factors affecting the costs associated with just handling and distribution. Normal replenishment of line-maintenance-station assembly inventories is accomplished by regularly scheduled flights from Atlanta. The cost associated with distributing these spares from the secondary inventory in Atlanta is largely a shipping cost, but its value depends on the shipping priority. If revenue cargo is displaced, the magnitude of the shipping cost will be greater than if revenue cargo is not displaced. Whether or not revenue cargo is displaced will depend upon how quickly the spare is needed, the quantity of revenue cargo ready to be shipped, the aircraft fuel requirements, the number of passengers on the flight, etc.

The inventory shortage costs appear as flight-delay and expediting-action costs. The determination of costs associated with flight delays is very difficult. Flight delay costs are similar to rate of return in an investment problem. The question is more one of what is the airline willing to spend to prevent a delay, than one of what the cost of the delay is (29). The actual cost of a flight delay is related to the length of the delay, the number of passengers and amount of cargo delayed, and the loss of goodwill caused by customer inconvenience.

The cost of expediting will vary depending upon the circumstances of the situation. The expediting cost may be only the loss of revenue cargo if there is a scheduled flight soon enough to transport the needed spare from Atlanta secondary stock or another station. However, the expediting cost may be very high if it is necessary to make a special flight to transport the needed spare.

The inventory model developed for assembly-stocking policies at maintenance bases and line maintenance stations is based on one of the models presented by Churchman, Ackoff and Arnoff (30). It was shown that the daily assembly demand for a station ($\bar{G}$) could be approximated by a Poisson distribution. Since a Delta maintenance official stated that station inventories are replenished daily, a constant replenishment lead time of one day was assumed to exist.

The probability of G assemblies being required in one day at a station is given by

$$P(G) = \frac{e^{-\bar{G}} \bar{G}^{G}}{G!}$$

The annual cost of carrying inventory at the station is given by

$$C_1 C_A \sum_{G=0}^{M} P(G)(M-G)$$

where C_1 is the 20 per cent inventory carrying cost, C_A is the total cost of an assembly, and M is the number of spares stocked at the station.

The annual inventory shortage cost at the station is given by

$$365\ C_2 \sum_{G+1}^{\infty} P(G)(G-M)$$

where C_2 is the cost associated with each inventory shortage, and the factor 365 is the number of days in a year.

The total expected annual cost is given by

$$TEC = 365\ C_2 \sum_{G+1}^{\infty} P(G)(G-M) + C_1 C_A \sum_{G=0}^{\infty} P(G)(M-G)$$

This TEC equation can be used to determine the value of M which will give the lowest annual cost. A Delta maintenance official estimated that the new wheel cost is \$245.00, the new tire cost is \$115.00, and the cost of mounting a tire upon a wheel is \$5.00. Summing these costs yields a C_A cost of \$365.00.

Two approaches were used to assign a value to the cost of an inventory shortage. It was found that in 1959 there were six instances of inventory shortages. All of these six shortages occurred at stations which did not stock any spare assemblies. In 1959 there were a total of 22 such stations that did not stock any spare assemblies. It was assumed that if each of these 22 stations had stocked one spare, none of the six shortages would have occurred. This was based on the fact that there were no instances of more than one shortage per station per day. The inventory carrying costs associated with stocking these 22 stations with one assembly is given by

$$22\ C_1C_A = \$1,606$$

Dividing this inventory carrying cost by the six instances of shortage yields a value of \$268. If this inventory carrying cost had been the only cost required to enable these 22 stations to perform assembly removals, it could be assumed that the airline did not feel that the cost of a shortage exceeded \$268. However, there would also have been the cost of additional maintenance personnel and facilities at several of these 22 stations. Since no study was made to determine the magnitude of these additional costs, it could only be concluded that the airline did not feel the cost of a shortage exceeded the amount of \$268 plus these additional costs.

A second approach to estimating C_2 is given by Churchman, Ackoff and Arnoff (31). An analytic solution is given to determine what range of values the decision-maker placed on a shortage. To illustrate this approach the 1959 Chicago Midway $\bar{G}$ and M are used. Estimates of C_2 were calculated using the $\bar{G}$ in Seasons I and III, which were 0.40 and 0.70, respectively. The M in 1959 was constant, and was equal to five spare assemblies. Estimates of C_2 can be obtained by solving the following inequality

$$P(G \leq M-1) < \frac{365\ C_2}{C_1C_A + 365\ C_2} < P(G \leq M)$$

In this inequality M = 5; C_1C_A = \$73; and the values of $P(G \leq M-1)$ and $P(G \leq M)$ were found in Molina's Tables (32) using the Season I value of $\bar{G}$ which was 0.40. Substituting in the above inequality gives

$$.9999388 < \frac{365\ C_2}{\$73 + 365\ C_2} < .9999960$$

The minimum value of C_2 is found by letting

$$\frac{365\ C_2}{\$73 + 365\ C_2} = .9999388$$

Solving this gives C_2 a minimum value of \$3,268. The maximum value of C_2 is found by letting

$$\frac{365\ C_2}{\$73 + 365\ C_2} = .9999960$$

Solving this gives C_2 a maximum value of \$50,000.

The same procedure is used for the Season III value of $\bar{G}$ which was 0.70. The minimum value of C_2 was found to be \$254 and the maximum value was found to be \$2,225. This analysis indicated that a seasonal change in $\bar{G}$ from 0.40 to 0.70 can cause significant changes in the range of values the decision-maker places upon C_2. If the decision-maker desires to place a constant range of values upon C_2 throughout the year, it will be necessary to adjust the inventory level M because of seasonal fluctuations in $\bar{G}$.

If the inventory level M were decreased in Season I to four, the above procedure would yield a minimum C_2 value of \$244 and a maximum value of \$3,268. These values are quite similar to the Season III minimum and maximum C_2 values for an M of five. This analysis shows that the decision-maker could reduce the inventory level M in Season I to four units and have nearly the same range placed upon C_2 as in Season III with an M of five units.

The following procedure may be used to establish the station inventory level M after the mean daily assembly demand $\bar{G}$ and a value for C_2 are estimated. The value of M which minimizes TEC is the value which satisfies the condition

$$P(G \leq M-1) < \frac{365\ C_2}{C_1 C_A + 365\ C_2} < P(G \leq M)$$

This value of M can be found by the following steps:

1. Prepare a table showing P(G) and $P(G \leq M)$ for each reasonable value of G.

2. Compute $\dfrac{365\ C_2}{C_1 C_A + 365\ C}$

3. Find the value of M which satisfies the above inequality.

The next model developed is the decision model for stocking policies at the Atlanta secondary inventory. This secondary inventory is used to replenish inventories at the line maintenance stations and the Atlanta maintenance base. In the model development it was assumed, for simplicity, that the Dallas maintenance base also had its inventory replenished from the secondary inventory. (This was not the true situation, since Dallas overhauls unserviceable wheels and sends the unserviceable tires to Atlanta for retreading. Serviceable tires are sent from Atlanta to Dallas where they are mounted on the overhauled wheels and then placed in the Dallas maintenance base inventory. See Figure 10.)

The inventory demand parameter (landings per removal) for the system has been extensively discussed in previous sections. A method of forecasting monthly L_i/R_i from t_i temperatures and two correction factors was developed. This forecasted monthly L_i/R_i divided into system monthly landings (L_i) will yield an estimate of the total monthly assembly demand on the secondary inventory. A Delta maintenance official stated that the secondary inventory is replenished irregularly, but that Delta policy insures that its level is adequate to replenish station inventories. It was assumed that the secondary inventory had a constant replenishment lead time of seven days. This lead time was used because the Delta official also stated that unserviceable tires are sent to the retreading agency approximately one time a week, and serviceable retreaded tires are returned at the time the unserviceable tires are sent. This lead time assumes that the serviceable retreaded tires and procured new tires are mounted on wheels and put into secondary stock every seven days. The actual procedure is for the serviceable tires to be mounted on wheels and put into secondary inventory at random. Although a constant seven-day lead time was used in the secondary inventory analysis, the same method of analysis could be followed for any other constant lead time period.

The following analysis of the secondary inventory will show how the forecasting procedure for L/R can be applied to adjust the secondary inventory level. The secondary inventory demand was forecasted for the period April 1957 through April 1960 by using the Delta $\bar{g}_i$ regression line equation in Table 33. This equation was used in lieu of a combination of the Delta $\bar{n}_i$ and Delta $\bar{r}_i$ regression equations in Table 33, because data on retreaded-tire utilization was available only for 1959. This $\bar{g}_i$ equation was

$$\bar{g}_i = 459.7 - 2.51\ t_i$$

The temperature factor t_i was estimated for the ith month by

$$t_i = \frac{m_i + q_{i-1}}{2}$$

where m_i is shown in Table 1 and q_{i-1} in Table 29.

The monthly forecasted values of $\bar{g}_i/4$ are shown in Table 53. This forecasted value was adjusted by the following correction factors:

1. $.21 - .357(m_i - q_{i-1})$
2. $.26 + .301 \sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$

The monthly values of $\sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$ are shown in Table 37.

The corrected forecast ($\bar{c}_i/4$) is shown in Table 53.

It was assumed that the number of landings in the ith month (L_i) could be accurately forecasted in the i-1 month. Table 53 shows the monthly forecasted number of assembly removals (Q_i), which was obtained by dividing the L_i in Table 34 by the $\bar{c}_i/4$ in Table 53. The actual R_i are shown in Table 53 also.

Since the replenishment lead time was assumed to be seven days, the forecasted mean weekly demand ($\bar{A}$) and actual mean weekly demand ($\bar{B}$) are shown in Table 54. It was shown that the daily system assembly demand could be approximated by the Poisson distribution; therefore, it was assumed that the weekly system assembly demand could also be approximated by the Poisson distribution. It was assumed that management would tolerate a probability of shortage of no more than one per cent, or less than one shortage every 100 weeks. The values of I_A necessary to satisfy the condition $P(A > I_A) < .01$ are shown in Table 54. The values of I_A were obtained from Molina's Tables (33) using the forecasted $\bar{A}$ in Table 54. The monthly values of I_B were obtained similarly, and are also shown in Table 54. It next was desirable to compare I_A and I_B. Figure 11 shows the monthly $\bar{A}$, the monthly I_A based on the forecasted $\bar{A}$, the monthly $\bar{B}$, and the monthly I_B based on the actual $\bar{B}$. It can be seen that the pattern of seasonal fluctuation for $\bar{A}$, $\bar{B}$, I_A, and I_B is similar. If the forecasted I_A were used to estimate I_B, the probability of a shortage would be

Table 53. Monthly Delta Forecasted Mean Service-Life ($\bar{g}_i/4$), Forecasted Adjusted Mean Service-Life ($\bar{c}_i/4$), Forecasted Number of Assembly Removals (Q_i), and Actual Number of Assembly Removals (R_i)

Month	$\bar{g}_i/4$	$\bar{c}_i/4$	Q_i	R_i
4/57	79.4	77.3	94.3	109
5/57	72.7	74.4	104.9	106
6/57	68.4	68.4	109.7	114
7/57	65.6	68.4	111.0	123
8/57	65.5	69.7	111.4	111
9/57	66.8	72.2	99.4	98
10/57	72.0	77.9	95.7	101
11/57	80.1	83.8	81.7	102
12/57	83.9	90.5	78.9	73
1/58	86.4	84.3	88.1	83
2/58	88.2	83.1	82.9	76
3/58	86.4	80.5	93.5	81
4/58	79.9	73.5	102.1	97
5/58	73.3	70.7	102.4	95
6/58	68.5	68.1	102.3	100
7/58	65.9	68.2	103.7	101
8/58	65.5	68.5	102.6	103
9/58	67.0	70.8	100.7	95
10/58	71.8	76.5	96.2	91
11/58	79.0	81.9	83.2	77
12/58	83.0	84.3	85.1	78
1/59	87.7	82.8	88.1	76
2/59	87.1	81.3	80.5	71
3/59	83.3	79.2	98.5	89
4/59	79.5	73.8	102.4	108
5/59	73.1	71.2	105.0	112
6/59	68.0	68.0	105.9	117
7/59	66.0	68.4	109.6	124
8/59	65.4	68.8	112.3	118
9/59	66.4	71.3	102.4	102
10/59	72.0	76.6	98.4	104
11/59	78.1	83.5	84.0	102
12/59	84.5	88.1	83.7	86
1/60	86.6	85.6	84.4	88
2/60	86.5	85.1	86.6	94
3/60	84.5	81.1	93.5	90
4/60	82.4	74.8	106.7	116

Table 54. Monthly Delta Actual Weekly Mean Assembly Demand ($\bar{B}$), Forecasted Weekly Mean Assembly Demand ($\bar{A}$), Actual Required-Assembly Inventory Level (I_B), Forecasted Required-Assembly Inventory Level (I_A), and the Probability that the Actual Weekly Demand is Greater than the Forecasted Required-Assembly Inventory Level $P(B > I_A)$

Month	$\bar{A}$	$\bar{B}$	I_A	I_B	$P(B > I_A)$
4/57	22.0	25.4	34	38	.034
5/57	23.7	23.9	36	36	.008
6/57	25.6	26.6	38	39	.017
7/57	25.1	27.8	38	41	.028
8/57	25.1	25.1	38	38	.006
9/57	23.2	22.9	35	35	.007
10/57	21.6	22.8	33	35	.019
11/57	19.0	23.8	30	36	.096
12/57	17.9	16.5	28	27	.004
1/58	19.9	18.8	31	30	.004
2/58	20.7	19.0	32	30	.002
3/58	21.1	18.3	33	29	.000
4/58	23.8	22.6	36	34	.004
5/58	23.1	21.4	35	33	.002
6/58	23.9	23.3	36	35	.004
7/58	23.5	22.8	36	35	.004
8/58	23.2	23.2	35	35	.007
9/58	23.5	22.2	36	34	.002
10/58	21.7	20.6	33	32	.006
11/58	19.4	18.0	30	29	.003
12/58	19.3	17.6	30	28	.003
1/59	19.9	17.2	31	28	.001
2/59	20.2	17.7	31	28	.002
3/59	22.3	20.1	34	31	.001
4/59	23.9	25.2	36	38	.015
5/59	23.7	25.3	36	38	.015
6/59	24.7	27.3	37	40	.026
7/59	24.7	28.0	37	41	.041
8/59	25.3	26.7	38	39	.017
9/59	23.9	23.8	36	36	.008
10/59	22.2	23.5	35	36	.010
11/59	19.6	23.0	31	35	.044
12/59	18.9	19.4	30	30	.007
1/60	19.0	19.9	30	31	.013
2/60	20.9	22.7	32	35	.029
3/60	21.1	20.3	33	32	.003
4/60	24.9	27.1	37	40	.026

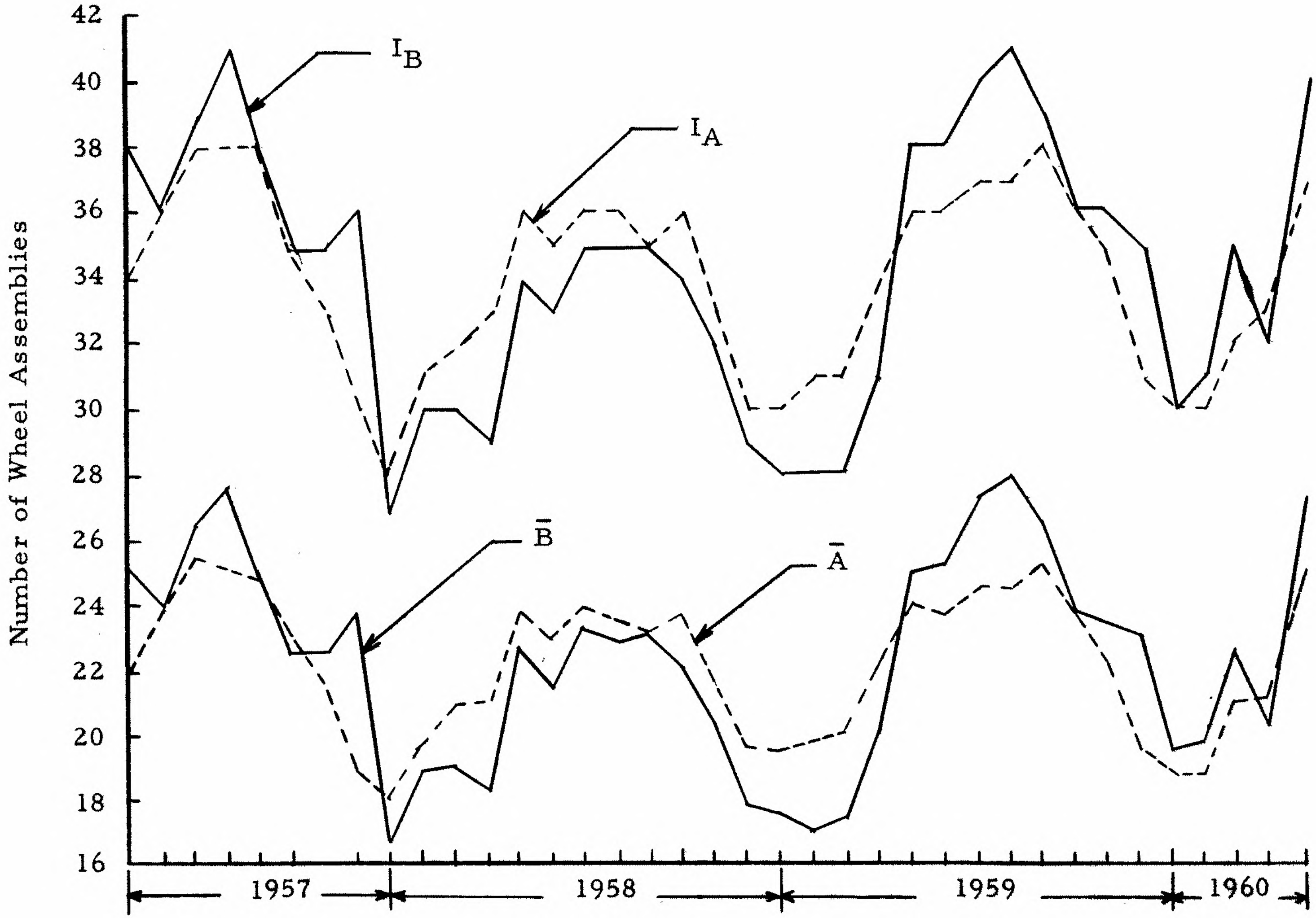


Figure 11. Monthly Variation of Delta Actual Weekly Mean Assembly Demand ($\bar{B}$), Forecasted Weekly Mean Assembly Demand ($\bar{A}$), Actual Required-Assembly Inventory Level (I_B), and Forecasted Required-Assembly Inventory Level (I_A)

$P(B > I_A)$. Table 54 shows the monthly values of $P(B > I_A)$. The average monthly value of $P(B > I_A)$ was found to be 0.014, which closely approximates the assumed probability of a shortage of 0.010 that management would tolerate. From this it was assumed that I_A could be used to estimate I_B.

This approach to the secondary inventory policy problem was used in lieu of the approach for station inventory policies, because of the difficulty in determining costs associated with a shortage in secondary inventory. An inventory shortage at a station will result in a flight delay, unless there is sufficient time after a flight termination to procure the needed spare. However, a shortage in secondary inventory will cause a flight delay only when a station has an inventory shortage due to its inventory not being replenished. This fact complicates the estimation of costs associated with shortage in secondary inventory, therefore, it was assumed the secondary inventory policy model would be based on a probability of shortage that management would tolerate, and a forecasted mean assembly demand for the inventory replenishment lead time. The forecasted mean assembly demand is obtained by following the developed forecasting procedure to calculate the quantity $\bar{c}_i/4$, which is used as an estimate of L_i/R_i. The number of landings occurring in the replenishment lead time period divided by the quantity $\bar{c}_i/4$ will yield an estimate of the mean assembly demand for the replenishment period. The equation for $\bar{c}_i/4$ is as follows:

$$\bar{c}_i/4 = \frac{459.7 - 2.51\,(m_i + q_{i-1})\,/2}{4} + .21 - .357\,(m_i - q_{i-1}) +$$

$$.26 + .301 \sum_i (\bar{g}_{i-1}/4 - L_{i-1}/R_{i-1})$$

The assembly inventory level which will satisfy the probability of a shortage to be tolerated can be obtained from Molina's Tables (34) using the forecasted mean assembly demand. This procedure provides a tool to adjust secondary inventory levels for changes occurring in replenishment lead times, seasonal temperature fluctuations, and variations in landing frequencies.

The third model considered is the decision model for tire-stocking policies at the Atlanta maintenance base. The total number of tires in inventory is considered to consist of the unserviceable tires at the retreading agency, the unserviceable tires which are being accumulated at the Atlanta maintenance base to be sent to the retreading agency, newly procured tires awaiting mounting, and the serviceable mounted tires in secondary inventory. As stated previously, unserviceable tires are sent to the retreading agency approximately once a week, and serviceable retreaded tires are returned at the same time the unserviceable tires are sent. The number of tires sent to the retreading agency depends upon how many of the weekly removals are in a condition to be retreaded. Some of the weekly tire removals will be scrapped because of permanent tire carcass damage, or because of the tire carcass having been retreaded the maximum number of times permitted by management policy. It was assumed that the proportion of tires scrapped is a constant proportion of the number of tires removed. This proportion was estimated by the data in Table 13, which shows that 405 new tires and 967 retreaded tires were among the removals made in the period April 1957 through April 1960. Since new tires are procured to replace scrapped tires, it was assumed that the proportion of tire removals that are scrapped is equal to

$$\frac{\text{Number of new tires}}{\text{Number of new tires + number of retreaded tires}} = 0.30$$

The total tire inventory (U) required to meet the seven day I_B replenishment lead time has the following components at the end of the lead time period:

1. $0.70\ \bar{B}$ serviceable tires returned from the retreading agency
2. $0.30\ \bar{B}$ procured new tires
3. $I_B - \bar{B}$ serviceable tires in secondary inventory
4. $0.70\ \bar{B}$ unserviceable tires sent to the retreading agency

This required inventory U assumes that on the morning of the seventh day of the lead time, the 0.70 $\bar{B}$ unserviceable tires are returned by the retreading agency to the Atlanta maintenance base in serviceable condition. It was further assumed that on the seventh day the 0.70 $\bar{B}$ retreaded tires and 0.30 $\bar{B}$ procured new tires are mounted on wheels and placed into secondary inventory. It was not necessary to consider the time required to overhaul wheels in estimating the I_B replenishment lead time, since $\bar{B}$ wheels can be overhauled in a much shorter period than is needed to retread $\bar{B}$ tires.

The control of new-tire procurement can be used to adjust the tire inventory levels as seasonal variations occur in the required inventory U. Monthly forecasts of U_i were calculated by using the forecasted $\bar{A}$ and I_A in Table 54 as estimates of $\bar{B}$ and I_B, respectively. The forecast of U_i was designated T_i, and is given by

$$T_i = 0.70\ \bar{A} + 0.30\ \bar{A} + (I_A - \bar{A}) + 0.70\ \bar{A} = I_A + 0.70\ \bar{A}$$

The monthly forecasts of T_i are shown in Table 55. Also shown in Table 55 are monthly forecasts of the number of tires scrapped (J_i) and monthly forecasts of the number of new tires to be procured $\left[J_i + (T_i - T_{i-1})\right]$. The forecasted values of J_i were obtained by multiplying the forecasted Q_i in Table 53 by the scrappage proportion of 0.30.

Figure 12 shows the monthly variations in the forecasted T_i and $\left[J_i + (T_i - T_{i-1})\right]$. No data was obtained relative to the actual number of new tires procured or the actual number of tires scrapped;

Table 55. Monthly Delta Forecasted Required-Tire Inventory Level (T_i), Forecasted Number of Scrapped Tires (J_i), Quantity ($T_i - T_{i-1}$), and Forecasted Number of New Tires to be Procured $\left[J_i + (T_i - T_{i-1})\right]$

Month	T_i	J_i	$T_i - T_{i-1}$	$J_i + (T_i - T_{i-1})$
4/57	49	28		
5/57	53	31	4	35
6/57	56	33	3	36
7/57	56	33	0	33
8/57	56	33	0	36
9/57	51	30	-5	25
10/57	48	29	-3	26
11/57	43	25	-5	20
12/57	41	24	-2	22
1/58	45	26	4	30
2/58	46	25	1	26
3/58	48	28	2	30
4/58	53	31	5	36
5/58	51	31	-2	29
6/58	53	31	2	33
7/58	52	31	-1	30
8/58	51	31	-1	30
9/58	52	30	1	31
10/58	48	29	-4	25
11/58	44	25	-4	21
12/58	44	26	0	26
1/59	45	26	1	27
2/59	45	24	0	24
3/59	50	30	5	35
4/59	53	31	3	34
5/59	53	32	0	32
6/59	54	32	1	33
7/59	54	33	0	33
8/59	56	34	2	36
9/59	53	31	-3	28
10/59	51	30	-2	28
11/59	45	25	-6	19
12/59	43	25	-2	23
1/60	43	25	0	25
2/60	47	26	4	30
3/60	48	28	1	29
4/60	54	32	6	38

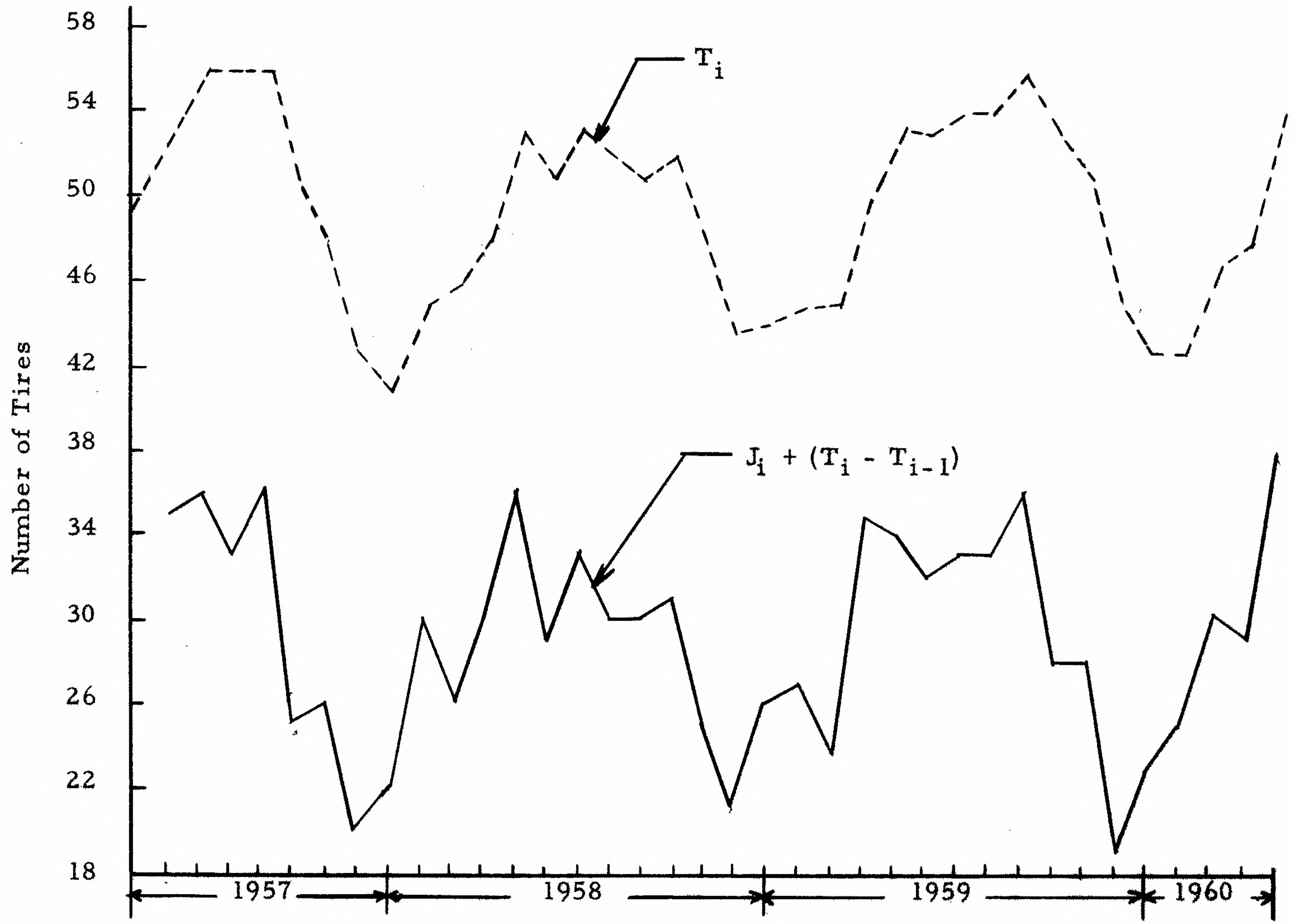


Figure 12. Monthly Variation of Delta Forecasted Required-Tire Inventory Level (T_i) and Forecasted Monthly Number of New Tires to be Procured $\left[J_i + (T_i - T_{i-1})\right]$

therefore, it was not possible to compare the forecasted $J_i + (T_i - T_{i-1})$ with the actual number of new tires procured.

The monthly variations in T_i are principally caused by the seasonal fluctuations in $\bar{A}$ and I_A. The decision-maker can use the forecasting procedures developed for $\bar{A}$ and I_A to forecast the level of T_i required. The level of T_i required can then be reached by procuring $J_i + (T_i - T_{i-1})$ new tires. Following this procedure would cause more new tires to be procured in the warmer months than in the cooler months. This would be true because $\bar{B}$, I_B, and R_i are all directly related to climatological temperature fluctuations.

The above discussions have shown how the forecasting methods developed for assembly service-life can be applied to make decisions relative to inventory stocking policies for:

1. Assemblies in line-maintenance-station and maintenance-base inventories
2. Assemblies in Atlanta secondary inventory
3. Tires at the Atlanta maintenance base

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions. --The results of this study are summarized in the following conclusions:

1. The retread stage (a) does not have a significant effect on $\bar{r}_a$ or s_a^2. This enables the use of $\bar{r}$ and s_r^2 as estimates of retreaded-tire service-life mean and variance, respectively.

2. Monthly retreaded-tire mean service-life ($\bar{r}_i$) is significantly greater than monthly new-tire mean service-life ($\bar{n}_i$).

3. The monthly mean service-life of new and retreaded tires is inversely related to climatological temperature fluctuations in t_i. This enables $\bar{n}_i$ and $\bar{r}_i$ to be forecasted from the known q_{i-1} and the m_i estimate of q_i.

4. The mean service-life of all monthly removals divided by four ($\bar{g}_i/4$) can be adjusted by two correction factors to provide a good estimate of the inventory demand parameter, landings per assembly removal (L/R).

5. The number of system daily landings (L) divided by $\bar{c}_i/4$ will yield an estimate of the system mean daily assembly demand. This daily assembly demand can be approximated by the Poisson distribution.

6. The L/R for the individual stations varies from station to station because of differences in the maintenance capability of stations, differences in the proportion of station landings resulting in flight terminations, and management policies. The station daily assembly demand can be approximated by the Poisson distribution.

7. Airline policy of prematurely removing assemblies reduces the number of flight delays.

8. Assembly removals causing flight delays are usually caused by assemblies becoming unserviceable early in their service-life.

9. Retreaded tires cause a greater proportion of flight delays than would be expected from the proportion of retreaded-tire utilization.

10. The proportion of new tires removed for wear is greater than the proportion of retreaded tires removed for wear.

11. Station assembly inventory levels (M) can be adjusted for seasonal variations in station mean daily assembly demand when values are established for the inventory carrying cost and the cost of inventory shortage. It was shown that the station daily assembly demand could be approximated by the Poisson distribution.

12. The probability of inventory shortage to be tolerated and the inventory replenishment lead time can be used with the Poisson distributed system daily assembly demand to establish the required secondary inventory level. The forecasted $\bar{c}_i/4$ and daily system landings can be used to estimate the seasonal temperature fluctuations in

the system mean daily assembly demand.

13. The tire inventory level can be adjusted for seasonal temperature fluctuations in system mean daily assembly demand by new-tire procurement policies. The level of monthly new-tire procurement can be taken as a function of monthly assembly removals, proportion of removed tires that are scrapped, and monthly changes in the system mean daily assembly demand.

Recommendations. --The recommendations for further study resulting from the assembly service-life and inventory model analyses of this study are as follows:

1. Assembly service-life factors to be studied for other aircraft types and aircraft positions (i. e., main, nose, and tail).

   a. The effect of the retread stage on retreaded-tire service-life.

   b. Comparison of new and retreaded-tire service-life.

   c. Correlation of monthly climatological temperatures and the monthly assembly mean service-life.

   d. Assembly service-life lost in following the policy of prematurely removing assemblies.

   e. Relationship between assembly service-life and landings per assembly removal.

2. Inventory policy.

   a. Shortage costs associated with station and secondary assembly inventory shortages.

   b. Effect of variable replenishment lead times on station and secondary inventory levels.

   c. Evaluation of inventory economies possible by monthly adjustment of assembly and tire inventories.

It is suggested that airlines employ the assembly service-life forecasting procedure developed in this study to develop service-life forecasting models for assemblies on other types of aircraft. The forecasting of assembly service-life can be used in conjunction with inventory adjustment and maintenance scheduling.

APPENDICES

APPENDIX A

Service-Life (in Landings) of Braniff Individual Removals Classified by New Tire and Retread Stage for Period January 1957 Through October 1959

New Tires

278	334	154	393	334	166	379	415
223	378	315	395	257	363	386	90
323	341	215	398	190	366	260	396
355	265	359	404	95	569	372	311
342	344	323	303	304	273	374	271
445	365	407	85	216	276	325	279
370	368	579	139	328	387	151	231
204	209	101	241	193	468	314	318
271	317	390	241	192	386	140	235
321	245	242	267	288	73	307	199
175	213	288	191	74	259	240	609

First Retread Stage

622	323	506	63	347	234	437	439
499	317	307	466	575	200	180	93
428	185	153	782	243	410	433	465
450	291	440	194	174	433	444	545
272	260	275	352	477	393	419	356
543	476	320	506	552	302	373	574
454	331	327	354	422	286	200	130
356	316	401	507	346	397	374	627
399	322	557	519	422	330	201	130
32	387	445	491	374	402	347	14
396	388	399	351	389	324	135	390
323	326	259	444	367	357	232	371
372	305	278	368	417	134	416	332
409	152	178	395	376	334	487	56
335	450	539	394	402	339	493	166
195	388	431	385	58	147	435	420
489	434	502	440	416	308	84	482
611	321	401	631	370	373	238	279

First Retread Stage (Continued)

292	283	333	287	375	196	228
294	305	337	30	292	193	287
348	287	463	344	231	326	254
389	334	386	374	391	109	

Second Retread Stage

464	416	211	323	448	335	320	329
381	210	351	550	638	429	415	323
526	270	223	383	374	390	416	291
396	346	359	444	337	386	9	341
371	285	329	222	576	447	362	334
23	430	463	686	508	458	278	355
10	185	355	427	353	477	418	365
375	188	379	366	230	473	412	236
373	307	468	446	400	434	135	193
209	25	343	409	460	434	354	0
447	50	417	206	435	463	299	160
256	350	200	261	135	558	426	249
362	340	510	524	24	436	363	268
328	346	459	461	19	159	398	397
447	433	344	469	277	305	281	300
271	381	408	474	393	459	316	351
213	159	553	373	79	287	217	244
416	433	469	248	338	219	427	428
333	327						

Third Retread Stage

153	71	274	397	171	427	496	100
530	395	155	523	74	157	387	510
351	266	387	420	460	247	397	513
264	540	242	639	102	311	391	323
265	517	412	473	482	150	299	560
393	414	384	377	219	297	235	455
201	302	356	424	325	344	251	581
257	81	77	394	424	437	233	209
310	478	359	151	395	58	458	322
226	390	402	468	241	451	380	480
325	229	70	477	334	293	365	225
195	140	132	423	292	347	191	279
64	447	356	527	320	376	296	319
143	416	431	487	3	398	524	352

Third Retread Stage (Continued)

411	403	317	323	434	304	321
243	222	421	315	390	104	264
333	283	68	180	388	147	284
245	285	60	367	329	258	

Fourth Retread Stage

71	263	273	350	340	476	421	306
548	167	215	449	361	227	163	394
490	332	325	331	324	523	344	288
267	231	365	387	394	255	288	446
341	399	539	211	266	553	215	363
347	216	381	212	334	270	451	367
525	472	264	307	115	502	418	382
244	356	54	382	308	558	60	52
381	315	357	233	455	332	411	394
408	315	431	346	49	427	298	194
302	58	123	284	275	438	323	252
227	418	321	209	344	149	435	372
271	102	67	315	334	414	402	33
426	443	301	161	154	367	289	233
382	352	444	261	284	386	310	394
565	401	415	73	342	117	200	

Fifth Retread Stage

403	309	277	413	182	136	145	361
505	407	343	381	309	493	258	
19	207	147	231	304	551	485	
378	222	444	448	57	568	446	
131	223	381	236	343	253	378	
142	451	249	359	352	198	353	
317	210	336	346	300	386	386	
414	357	474	227	233	75	262	

Sixth Retread Stage

240	241	433	321	433	270	445
107	312	379	174	288	188	476
163	469	213	385	265	347	338
365	409	66	309	391	311	
94	513	132	317	416	385	

APPENDIX B

Service-Life (in Hours) of Delta Removals Classified by Months for the Periods April 1957 Through December 1958 and January 1960 Through April 1960

April 1957

208	273	262	138	180	302	302	404
194	309	399	386	347	171	13	281
27	384	274	320	61	324	300	260
229	228	308	241	418	317	483	200
288	185	163	225	314	216	249	183
239	473	349	247	172	299	425	149
239	145	180	142	401	318	438	310
292	226	236	359	287	349	289	270
337	305	328	181	324	376	227	286
265	125	386	323	415	426	313	404
283	394	357	362	365	263	325	287
173	266	168	353	271	14	290	
247	416	269	384	56	331	339	
298	267	384	375	404	429	172	

May 1957

326	222	238	298	406	188	262	181
253	203	378	232	296	259	359	211
119	214	263	302	155	226	138	222
265	203	325	350	315	194	280	152
296	387	290	236	353	10	292	192
329	250	252	10	268	30	313	206
232	255	266	279	297	323	242	310
171	162	305	201	307	425	339	287
217	204	371	209	261	168	491	
334	292	366	289	128	387	213	
266	240	239	235	239	222	164	
226	200	359	275	258	290	271	
193	311	292	251	195	243	210	
215	119	278	251	226	272	298	

June 1957

236	220	230	224	339	300	251	335
306	84	366	308	13	376	259	236
82	285	213	290	313	251	273	295
250	127	276	222	108	251	313	122
192	241	259	304	368	290	270	113
21	302	235	372	346	260	351	283
283	279	242	333	363	339	260	268
315	105	246	275	318	298	187	293
269	202	247	373	330	212	136	273
179	273	331	330	354	239	4	
193	287	317	91	210	116	39	
362	240	336	318	244	262	329	
187	255	311	326	299	59	225	
178	260	179	236	287	334	299	
194	39	265	291	325	271	382	

July 1957

284	125	314	40	75	208	258	212
301	304	264	250	297	206	252	218
294	261	7	139	227	235	240	277
268	215	55	188	74	149	208	222
241	346	250	334	249	63	258	170
412	234	264	264	346	345	207	303
247	43	276	315	247	235	118	276
81	310	140	315	266	242	202	276
34	209	68	438	282	236	127	248
195	212	117	328	302	249	314	282
246	301	311	316	287	165	264	279
44	275	291	374	275	220	179	
215	228	192	216	279	303	267	
220	187	278	267	64	240	122	
191	331	224	281	279	196	212	
358	175	349	5	249	246	216	

August 1957

171	253	147	271	247	281	251	207
181	328	267	234	305	345	267	137
161	309	358	266	176	206	252	157
333	277	233	266	228	283	212	137
181	295	211	320	50	300	268	147
298	243	190	307	519	200	118	287

August 1957 (Continued)

323	287	213	103	137	178	332	277
190	177	258	223	258	201	344	252
300	150	245	134	158	287	247	167
246	264	179	136	208	188	252	302
248	193	254	328	298	257	266	250
339	358	219	256	180	283	285	296

September 1957

369	310	253	253	286	297	269	259
239	273	98	228	222	303	282	245
279	167	285	263	187	273	249	272
330	211	319	323	251	241	328	292
190	10	158	326	168	246	201	356
264	300	267	272	322	260	120	253
201	478	40	246	273	280	433	335
300	220	278	273	172	97	199	361
193	298	352	267	298	249	330	232
257	178	273	251	198	64	263	254
284	223	43	337	288	329	230	276
289	203	232	285	188	288	297	495
259	133						

October 1957

291	305	245	340	393	222	188	171
190	148	137	203	213	305	259	298
267	191	207	226	247	249	219	259
303	350	311	119	389	418	289	117
255	284	313	397	218	376	308	335
281	278	89	90	294	247	300	193
223	317	373	232	117	341	87	200
281	221	222	268	238	253	365	187
206	208	336	337	246	276	173	250
272	206	277	149	235	358	215	232
328	293	340	327	269	234	99	293
360	296	343	285	323	285	410	61
207	177	366	165	286			

November 1957

274	437	356	356	318	286	206	365
324	291	128	224	258	216	243	230
275	184	202	256	306	292	241	238
303	224	262	247	317	279	316	295
242	238	283	203	304	334	56	231
186	280	283	83	44	128	284	220
261	247	330	22	263	268	227	72
415	352	106	324	325	272	289	313
291	286	270	256	233	351	330	220
235	311	336	67	276	303	265	288
291	238	220	241	315	260	296	278
298	272	277	325	176	226	224	408
191	246	247	359	255	262		

December 1957

103	228	228	342	369	385	79	127
230	256	313	256	201	247	118	289
295	366	240	271	283	298	254	295
306	249	239	254	278	204	249	296
226	257	258	260	362	273	300	211
377	373	385	244	313	265	313	307
222	288	320	251	289	110	158	282
288	337	162	235	369	260	287	346
337	321	240	258	269	243	256	245
205							

January 1958

370	256	255	286	362	300	300	280
262	361	185	416	8	323	394	313
349	288	337	311	354	197	262	408
280	277	294	318	455	242	310	395
325	238	404	330	321	333	256	409
384	301	360	66	294	411	411	156
237	266	382	270	326	439	425	380
276	391	361	321	346	360	427	394
369	363	291	253	306	307	214	340
398	400	411	379	281	384	61	290
144	349	322					

February 1958

451	478	317	369	339	232	262	324
245	316	267	270	334	483	340	299
344	432	253	233	206	404	305	356
569	638	241	250	330	217	351	369
386	318	217	420	239	273	287	247
175	386	346	306	379	277	463	276
312	189	216	253	270	245	452	501
454	463	334	438	359	362	454	368
349	334	322	138	430	270	384	428
316	399	282	399				

March 1958

0	383	464	341	282	275	251	341
350	336	412	284	299	323	333	273
296	346	260	56	81	353	143	243
302	350	290	298	327	126	412	343
39	272	206	274	145	350	329	345
281	452	221	257	181	338	424	407
380	430	152	353	393	495	45	464
415	438	366	367	328	382	293	456
414	398	49	309	352	384	280	225
198	345	425	292	390	232	175	373
436							

April 1958

239	313	94	356	463	240	212	107
293	308	289	372	298	345	351	249
291	172	317	329	334	286	346	276
313	301	282	356	232	215	253	264
375	250	281	434	199	360	366	359
208	244	191	285	425	353	338	369
455	366	292	322	263	369	264	242
348	225	316	316	180	482	423	267
255	397	240	351	97	342	359	304
320	342	272	312	362	294	301	280
373	105	181	154	332	281	274	329
260	289	309	347	177	316	378	319
188							

May 1958

212	282	228	299	225	358	325	348
332	334	166	299	258	236	125	280
382	233	260	314	296	224	345	256
270	253	352	397	264	343	218	186
319	258	485	251	298	311	364	262
221	0	320	287	430	293	320	336
269	288	341	197	124	120	325	287
340	407	394	272	294	197	284	262
277	351	329	288	273	250	220	313
320	359	354	293	292	188	181	401
246	210	298	234	253	289	265	300
140	283	202	286	280	308	322	

June 1958

233	294	260	254	209	186	294	250
363	183	299	231	255	285	258	196
193	341	312	196	273	247	192	259
288	225	191	280	329	249	248	255
275	231	280	338	294	242	224	222
285	362	130	266	294	363	196	198
324	347	288	267	308	330	284	273
249	258	239	253	247	305	253	331
258	297	250	238	291	237	195	207
251	305	155	341	191	229	271	98
309	258	128	169	256	265	184	184
133	240	298	234	274	313	267	216
262	267	135	223				

July 1958

227	338	292	68	194	250	342	195
249	235	211	222	246	261	262	299
240	255	92	218	259	266	276	279
191	266	270	251	29	244	250	320
276	269	263	175	268	289	111	138
238	307	247	269	293	276	243	258
245	210	294	293	205	163	278	249
240	243	96	241	213	384	241	278
290	177	265	218	230	156	224	264
278	244	251	275	271	236	264	78
215	254	219	279	241	255	120	196
275	261	198	267	323	268	206	191
247	316	134	320	188			

August 1958

71	267	239	291	246	249	228	226
12	303	295	290	208	279	325	204
6	174	247	37	267	103	193	176
287	437	319	250	189	261	264	262
136	312	192	166	244	238	237	176
247	306	268	0	121	253	337	232
322	171	204	232	122	152	232	180
221	164	192	236	314	193	241	242
338	269	281	147	210	300	183	260
329	344	154	220	258	262	254	250
218	283	263	275	235	317	223	184
227	247	211	176	87	154	188	204
260	247	210	241	151	203	267	

September 1958

296	364	82	325	144	173	195	99
204	262	28	301	195	233	265	301
369	220	330	253	239	419	252	244
405	293	283	373	139	259	247	315
279	266	245	333	308	276	266	211
150	131	151	241	172	233	359	301
246	268	239	376	265	217	306	209
175	250	316	319	350	305	250	286
241	218	237	303	243	135	0	80
232	263	184	290	192	299	334	272
252	221	274	252	276	258	141	225
262	281	200	362	328	235	280	

October 1958

300	282	282	253	199	240	158	302
270	264	323	305	236	272	299	316
393	273	358	374	204	424	169	102
299	205	346	182	168	288	212	310
259	191	203	286	211	295	292	276
296	279	259	261	282	276	188	367
340	276	260	292	324	261	261	194
199	215	282	282	134	105	374	279
312	238	398	388	271	307	319	238
295	299	264	370	287	295	324	237
171	344	331	170	229	181	113	288
307	294	234					

November 1958

115	249	382	286	171	171	302	410
379	133	204	347	330	358	359	253
175	312	331	358	277	229	355	349
184	359	272	251	399	345	258	262
299	332	219	247	263	269	269	404
385	282	302	257	278	180	276	289
284	189	272	203	455	358	402	334
397	368	26	307	361	331	253	226
315	358	301	296	275	279	327	239
241	279	329	2	279			

December 1958

262	314	405	403	257	310	258	352
320	333	376	334	392	270	450	315
352	392	144	407	241	454	366	268
369	457	245	413	129	147	293	272
3	335	328	346	406	291	312	432
359	252	414	373	360	362	499	477
436	415	390	327	371	180	403	202
210	31	12	266	293	302	392	314
327	253	344	106	328	408	322	303
72	336	289	413	154	362		

January 1960

406	376	120	95	354	191	353	232
268	372	242	338	258	218	350	256
286	371	339	303	348	264	295	317
267	193	247	272	421	289	316	236
256	377	399	338	312	406	350	335
307	457	339	290	454	280	406	283
291	341	384	320	303	302	376	385
306	263	236	344	344	324	359	287
257	280	393	352	394	303	316	89
274	379	405	365	342	387	321	267
304	372	21	287	329	264	276	449

February 1960

107	130	344	453	148	329	293	176
382	259	275	291	242	267	256	189
241	319	333	266	338	338	380	353
339	300	266	289	266	363	262	290
185	358	294	280	298	211	297	307
360	349	335	480	289	306	294	313
269	328	282	329	387	318	99	249
236	395	169	347	285	234	346	315
161	299	278	284	361	320	225	254
312	273	361	314	301	275	318	191
307	278	255	8	281	180	406	245
90	371	277	296	395	321		

March 1960

375	285	301	302	259	289	243	330
271	242	201	361	311	46	340	349
37	283	341	355	358	288	318	329
323	323	161	200	261	254	295	275
322	260	318	300	353	343	256	243
300	301	286	342	361	324	378	331
356	247	276	132	269	244	348	353
296	290	254	414	169	326	7	192
312	380	354	307	411	418	317	306
272	263	461	382	297	233	381	295
9	337	315	291	383	128	39	294
268	148						

April 1960

262	291	321	311	230	209	258	283
181	284	309	424	232	277	262	186
229	373	218	337	304	238	181	254
264	205	125	247	297	266	279	334
137	235	287	248	225	134	251	282
241	221	182	284	347	256	349	364
359	317	261	140	259	231	262	8
312	322	223	201	233	225	245	198
0	193	183	188	266	210	236	313
292	211	329	211	331	334	414	221
260	165	350	203	194	259	353	234
213	177	297	388	100	182	136	257
222	287	270	134	23	42	13	191
75	226	256	381	248	250	317	345
266	340	337	333				

APPENDIX C

Service-Life (in Hours) of Delta Removals Classified by Months, New Tire, and Retread Stage for the Period January 1959 Through December 1959

Month	New Tires	Retread Stage						Unknown
		First	Second	Third	Fourth	Fifth	Sixth	
1/59	289	524	336	460	394	399		382
	321	426	240	443	425	146		622
	287	526	130	352	464	485		
	361	157	280	241	407	223		
	214	347	135	459	437	447		
	302	386	219	365	514			
	214	433	546	382	442			
	272	434	426	495	501			
	302	379	271	66				
	390	346	502	152				
	441	379	390	142				
		335	488	451				
		627	182	190				
		328	182	250				
		405	390					
			145					
			336					
			504					
			320					
			553					
			238					
2/59	119	449	416	196	354	379	278	386
	378	448	434	491	349	385	243	294
	449	452	456	416	356	512	454	
	417	451	115	316	449	291		
	232	497	361	251	312	421		
	179	357	666	377	238			
	216	472	517	225	437			
	360	424	511	483	58			
	359		460	469	94			
	345		325	211				

Month	New Tires	Retread Stage: First	Second	Third	Fourth	Fifth	Sixth	Unknown
2/59	297		420	433				
(Cont.)	301		397	478				
	247			491				
	385			362				
				276				
				191				
				454				
				362				
3/59	276	406	403	285	302	390	67	
	415	466	212	516	255	516	469	
	280	331	354	368	305	354	103	
	253	8	385	188	488	588	334	
	348	340	380	346	405	0		
	283	314	468	350	216	318		
	298	444	317	165	214			
	219	363	393	358	239			
	215	359	123	359	449			
	163	418	248	418	240			
	247	448	425	363	390			
	220	288	319	413				
	220	403	325	352				
	189	270	367	287				
	269	85	257					
	184	424	372					
	104		353					
	246		479					
			263					
			251					
4/59	249	330	411	289	291	261		268
	280	446	347	345	367	189		
	232	270	292	310	249	289		
	257	150	451	342	284			
	250	357	291	330	341			
	210	108	337	340	327			
	216	367	331	228	188			
	211	300	424	224	232			
	272	123	277	204				
	241	376	264	276				
	289	278	304	344				
	270	248	258	206				

Month	New Tires	Retread Stage: First	Second	Third	Fourth	Fifth	Sixth	Unknown
4/59	165	258		377				
(Cont.)	301	417		167				
	213	311		344				
	195	266		181				
	265	308		361				
	215	334						
	13	217						
	260	481						
	240	246						
	237	252						
	244	322						
	225	107						
	180							
	254							
	249							
	257							
	277							
	259							
	243							
	273							
	252							
	237							
	266							
	298							
	229							
	296							
	274							
	300							
	258							
	263							
	258							
5/59	258	169	230	213	200	316	283	
	302	229	240	287	217	66		
	145	323	169	267	172	242		
	250	317	294	244	52	123		
	221	215	307	289	131			
	242	285	228	254	327			
	243	304	191	290	204			
	234	293	195	277	379			
	247	305	251	234	220			
	275	262	315	108	214			

	New	Retread Stage						
Month	Tires	First	Second	Third	Fourth	Fifth	Sixth	Unknown
5/59	228	286	279	215	286			
(Cont.)	351	322	281	245	215			
	192	276	91	264	171			
	208	292	235	352	209			
	259	254	278	241	386			
	256	107		255				
	184	273		354				
	111	367		265				
	298	340		266				
	303	188		219				
	258	359		283				
	282	385						
	207	290						
	145	259						
		22						
		217						
		378						
		398						
		262						
		392						
		264						
		201						
6/59	263	240	242	281	361	242	234	236
	202	390	281	254	280	306		
	210	296	246	402	426	88		
	209	263	256	344	236	44		
	325	304	267	246	314			
	237	228	394	230	187			
	220	310	221	328	267			
	206	278	12	259	245			
	244	326	69	91	278			
	129	205	248	284	264			
	217	368	251	183	287			
	244	341	183	261				
	193	222	259	247				
	68	228	121	78				
	318	231	346	308				
	153	277						
	247	249						
	247	285						
	249	194						

	New	Retread Stage						
Month	Tires	First	Second	Third	Fourth	Fifth	Sixth	Unknown
6/59	288	148						
(Cont.)	243	232						
	0	142						
		422						
		271						
		230						
		273						
		222						
		196						
		283						
		191						
		258						
		240						
		240						
		4						
		321						
		199						
		304						
		271						
		255						
		278						
		203						
		269						
		83						
		83						
		265						
		251						
		210						
		183						
7/59	209	331	78	304	132	281	185	363
	124	210	206	240	270	168		
	223	306	278	144	98	99		
	233	220	110	146	110	171		
	235	241	99	299	257			
	188	284	135	253	189			
	224	261	360	225	42			
	223	267	236	157	301			
	229	299	298	154	208			
	130	300	179	70	270			
	152	236	190	247	315			
	184	184	121	49	240			

Month	New Tires	Retread Stage: First	Second	Third	Fourth	Fifth	Sixth	Unknown
7/59	187	259	250	241	138			
(Cont.)	183	325	187	219	317			
	119	14	234		279			
	181	240	243					
	237	265	283					
	111	185	245					
	149	247	203					
	213	243	270					
	202	94	274					
	176	214	349					
	195	284	252					
	264	146	254					
	207	300	246					
	261	60	296					
	222	165	206					
		245	281					
		303	264					
		310						
		268						
		246						
		270						
8/59	244	199	282	235	217	242	204	100
	198	329	97	228	148	79	56	
	265	236	266	305	173	226	306	
	237	213	121	63	281	223		
	197	221	227	101	260	259		
	180	184	262	251	152	233		
	223	252	335	285	150	323		
	214	209	262	250	277			
	238	309	208	237	123			
	203	291	368	411	167			
		268	234	235	241			
		273	275	185				
		248	216	258				
		47	344	208				
		244	252	256				
		308	250					
		237	21					
		196	260					
		317	236					
		205	296					

Month	New Tires	Retread Stage						Unknown
		First	Second	Third	Fourth	Fifth	Sixth	
8/59 (Cont.)		248	220					
		247	197					
		230	297					
		220	200					
		246	258					
		240	269					
		226	230					
		216	229					
		83	211					
		99	307					
		264	131					
		234	319					
		325	269					
		296	276					
		292	296					
			215					
9/59	272	198	256	292	242	183	190	227
	268	231	176	256	321	283		310
	192	329	209	166	401	306		
	232	284	255	226	221	298		
	294	284	251	220	355	232		
	200	287	205	77	221			
	249	277	224	215	248			
	188	278	196	223	333			
	288	272	226	117	202			
	221	209	267	282	130			
	249	182	242	194				
	295	199	52	250				
	282	317	256	290				
	334	250	223	214				
	232	325	352	301				
		312	238	152				
		125	272	209				
		211	281	237				
		295	250	259				
		300	123	259				
		260	237					
			167					
			230					
			92					
			298					

Month	New Tires	Retread Stage						Unknown
		First	Second	Third	Fourth	Fifth	Sixth	
9/59 (Cont.)			330					
			277					
			278					
10/59	272	349	272	193	214	213	142	
	242	247	64	331	241	158	293	
	123	260	342	167	91		257	
	235	265	293	311	322			
	337	280	224	205	245			
	308	328	261	270	178			
	327	266	330	231	154			
	277	311	209	215	282			
	254	221	284	259				
	262	341	280	256				
	296	302	227	286				
	369	277	215	212				
	325	350	304	246				
	276	331	236	249				
	11	353	189	244				
	260	338	333	237				
	297	387	241	190				
	245	362	292	249				
	302	374	267	307				
	210	331	248	252				
	264	224	253					
		248	283					
		270						
		294						
		293						
		267						
		267						
		326						
11/59	270	228	339	331	223	278	255	
	238	236	311	207	248	150		
	290	294	217	293	242	39		
	223	296	210	248	314	210		
	236	243	206	272	267	233		
	74	232	48	245	272	300		
	290	342	289	259	256	339		
	196	383	365	26	300			
	290	272	290	256				

Month	New Tires	First	Second	Third	Fourth	Fifth	Sixth	Unknown
		Retread Stage						
11/59	241	246	343	292				
(Cont.)	320	311	220	336				
	94	232		309				
	317	285		361				
	276	42		352				
	310	284		156				
	289	307		233				
	279	209		292				
	329	206		234				
	295	379		311				
	273	265		219				
		221		88				
		208		257				
		253		208				
		289		221				
		231		285				
				305				
				247				
				294				
				261				
				307				
12/59	251	261	253	197	226	318	352	350
	392	246	329	189	368	208	203	
	285	243	287	8	311	367	225	
	298	261	263	255	83		194	
	221	255	291	252	239			
		299	328	266	170			
		166	352	329	324			
		278	321	206	233			
		194	323	135	277			
		268	303	267	202			
		277	350	280	216			
		248	333	263	216			
		305	344	315	204			
		356	310	364	305			
		262	399	248	323			
		297	427	295	297			
		288	283		366			
		317			236			
					253			
					229			
					305			
					289			

BIBLIOGRAPHY

Literature Cited

1. Military Specification for Casings, Tire and Tubeless Tires; Aircraft Pneumatic, MIL-C-5041B, 25 November 1953, United States Department of Defense, p. 37.

2. Murphy, Joseph S., "Jets to Spark Boom in Tire Retreading," American Aviation, (June 3, 1957), p. 57.

3. Loc. cit.

4. "KLM's Experience with Tire Recapping," American Aviation, (August 3, 1951), p. 51.

5. Murphy, op. cit., p. 57.

6. Ericksen, W. P., Landing-Gear and Tire Experience with the Boeing 707, Society of Automotive Engineers.

7. Murphy, op. cit., p. 58.

8. Vazsonyi, A., Scientific Programming in Business and Industry, New York, John Wiley and Sons, Inc., 1958, pp. 345-346.

9. Arrow, K. J., S. Karlin, and H. Scarf, Studies in the Mathematical Theory of Inventory and Production, Stanford, California, Stanford University Press, 1958, p. 16.

10. B. F. Goodrich Engineering Data on Aircraft Tires, B. F. Goodrich Aviation Products, Akron, Ohio, pp. 11-12.

11. "Aircraft Tires, Wheels and Brakes," Aircraft Accident and Maintenance Review, United States Air Force (April 1960), p. 2.

12. B. F. Goodrich Engineering Data on Aircraft Tires, B. F. Goodrich Aviation Products, Akron, Ohio, p. 12.

13. Hoel, P. G., Introduction to Mathematical Statistics, New York, John Wiley and Sons, Inc., 1954, pp. 172-175.

14. Duncan, A. J., Quality Control and Industrial Statistics, Homewood, Illinois, Richard D. Irwin, Inc., 1959, pp. 492-495.

15. Ibid., pp. 607-608.

16. Ibid., pp. 450-454.

17. Ibid. pp. 538-542.

18. Ibid., pp. 481-484.

19. Ibid., pp. 476-478.

20. Ibid., pp. 470-473.

21. Bowker, A. H., and G. J. Lieberman, Engineering Statistics, Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1959, p. 286.

22. Ireson, W. G. and E. L. Grant, Handbook of Industrial Engineering and Management, Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1955, pp. 933-944.

23. Bowker, op. cit., pp. 262-273.

24. Ibid., pp. 273-274.

25. Ibid., pp. 370-371.

26. Molina, E. C., Poisson's Exponential Binomial Limit, New York, D. Von Nostrand Company, Inc., 1949, Table 1, pp. 1-6.

27. Duncan, op. cit., pp. 467-469.

28. Whitin, T. M., The Theory of Inventory Management, Princeton, New Jersey, Princeton University Press, 1957, pp. 219-221.

29. Sanford, Thomas B., A Discussion of Inter-Airline Spare Parts Pooling, Unpublished paper presented to the Fourth Annual Air Transport Association Sub-Committee on Production Planning and Control.

30. Churchman, C. W., R. L. Ackoff, and E. L. Arnoff, Introduction to Operations Research, New York, John Wiley and Sons, Inc., 1957, pp. 207-211.

31. Churchman, op. cit., pp. 211-212.

32. Molina, op. cit., Table II, p. 2.

33. Molina, op. cit., Table II, pp. 19-22.

34. Molina, op. cit., Table II, pp. 1-47.